

# Nucleation of superconductivity and vortex matter in superconductor – ferromagnet hybrids

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**Abstract.** The theoretical and experimental results concerning the thermodynamical and low-frequency transport properties of hybrid structures, consisting of spatially-separated conventional low-temperature superconductor (S) and ferromagnet (F), is reviewed. Since the superconducting and ferromagnetic parts are assumed to be electrically insulated, no proximity effect is present and thus the interaction between both subsystems is through their respective magnetic stray fields. Depending on the temperature range and the value of the external field  $H_{ext}$ , different behavior of such S/F hybrids is anticipated. Rather close to the superconducting phase transition line, when the superconducting state is only weakly developed, the magnetization of the ferromagnet is solely determined by the magnetic history of the system and it is not influenced by the field generated by the supercurrents. In contrast to that, the nonuniform magnetic field pattern, induced by the ferromagnet, strongly affect the nucleation of superconductivity leading to an exotic dependence of the critical temperature  $T_c$  on  $H_{ext}$ . Deeper in the superconducting state the effect of the screening currents cannot be neglected anymore. In this region of the phase diagram various aspects of the interaction between vortices and magnetic inhomogeneities are discussed. In the last section we briefly summarize the physics of S/F hybrids when the magnetization of the ferromagnet is no longer fixed but can change under the influence of the superconducting currents. As a consequence, the superconductor and ferromagnet become truly coupled and the equilibrium configuration of this “soft” S/F hybrids requires rearrangements of both, superconducting and ferromagnetic characteristics, as compared with “hard” S/F structures.

Some figures in this paper are in color only in the electronic version.

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**List of main notations***Acronyms*

GL	Ginzburg-Landau,
DWS	domain-wall superconductivity,
F	ferromagnet or ferromagnetic,
OP	order parameter,
RDS	reverse-domain superconductivity,
S	superconductor or superconducting
1D	one-dimensional
2D	two-dimensional

*Latin letters*

<b>A</b>	vector potential, corresponding to the total magnetic field: $\mathbf{B} = \text{rot } \mathbf{A}$ ,
<b>a</b>	vector potential, describing to the nonuniform component of the magnetic field, $\mathbf{b} = \text{rot } \mathbf{a}$ ,
<b>B</b>	total magnetic field: $\mathbf{B} = \mathbf{H}_{ext} + \mathbf{b}$ ,
<b>b</b>	nonuniform component of the magnetic field induced by ferromagnet,
$c$	speed of light,
$D_s$	thickness of the superconducting film,
$D_f$	thickness of the ferromagnetic film (or single crystal),
$f$	absolute value of the normalized OP wave function: $f = \sqrt{(\text{Re } \psi)^2 + (\text{Im } \psi)^2}$ ,
$\mathbf{j}_{ext}$	the density of the external current: $\text{rot } \mathbf{H}_{ext} = (4\pi/c) \mathbf{j}_{ext}$ ,
$\mathbf{j}_s$	the density of superconducting currents,
$G_{sf}$	free (Gibbs) energy of the S/F hybrid,
$G_m$	term in the free energy functional accounting for the spatial variation of the magnetization,
$\mathbf{H}_{ext}$	external magnetic field,
$\mathbf{H}_{ex}$	exchange field,
$H_{c1}$	lower critical field: $H_{c1} = \Phi_0 \ln(\lambda/\xi)/(4\pi\lambda^2)$ ,
$H_{c2}$	upper critical field: $H_{c2} = \Phi_0/(2\pi\xi^2) = H_{c2}^{(0)} (1 - T/T_{c0})$ ,
$H_{c2}^{(0)}$	upper critical field at $T = 0$ : $H_{c2}^{(0)} = \Phi_0/(2\pi\xi_0^2)$ ,
$h$	separation between superconducting and ferromagnetic films,
$L$	angular momentum of Cooper pairs (vorticity): $\psi = f(r) e^{iL\varphi}$ ,
$\ell_H$	magnetic length: $\ell_H = \sqrt{\Phi_0/(2\pi H_{ext} )}$ ,
$\ell_b^*$	effective magnetic length determined by a local magnetic field $b_z^*$ : $\ell_b^* = \sqrt{\Phi_0/(2\pi b_z^* )}$ ,
$\ell_\psi$	typical width of the localized OP wave function,
<b>M</b>	magnetization of the ferromagnet,
$M_s$	magnetization of the ferromagnet in saturation,
$\mathbf{m}_0$	dipolar moment of a point-like magnetic particle,
$R_s$	radius of the superconducting disk,
$R_f$	radius of the ferromagnetic disk-shaped dots,
$\mathbf{R}_d$	position of a point-magnetic dipole: $\mathbf{R}_d = \{X_d, Y_d, Z_d\}$ ,
$T_{c0}$	superconducting critical temperature at $B = 0$ ,
$w$	period of the one-dimensional domain structures in ferromagnet

*Greek letters*

$\alpha, \beta$	constants of the standard expansion of the density of the free energy with respect to $ \Psi ^2$ ,
$\epsilon_v^{(0)}$	self-energy of the vortex line per unit length: $\epsilon_v^{(0)} = (\Phi_0/4\pi\lambda)^2 \ln \lambda/\xi$ ,
$\Theta$	the OP phase: $\Theta = \arctan(\text{Im } \psi / \text{Re } \psi)$ ,
$\lambda$	temperature-dependent magnetic field (London) penetration length: $\lambda = \lambda_0 / \sqrt{1 - T/T_{c0}}$ ,

$\lambda_0$	magnetic field penetration length at $T = 0$ ,
$\xi$	temperature-dependent superconducting coherence length: $\xi = \xi_0/\sqrt{1 - T/T_{c0}}$ ,
$\xi_0$	Ginzburg-Landau coherence length at $T = 0$ ,
$\pi$	3.141592653...,
$\rho$	electrical resistivity,
$\Phi_0$	magnetic flux quantum: $\Phi_0 = \pi\hbar c/e \simeq 2.07 \text{ Oe}\cdot\text{cm}^2$ ,
$\Psi$	superconducting order parameter (OP) wave function,
$\Psi_0$	OP saturated value, $\Psi_0 = \sqrt{-\alpha/\beta}$ ,
$\psi$	normalized OP wave function, $\psi = \Psi/\Psi_0$

### *Coordinate systems*

Throughout this paper we use both cartesian reference system  $(x, y, z)$  and cylindrical reference system  $(r, \varphi, z)$ , where  $z$ -axis is always taken perpendicular to the superconducting film/disk.

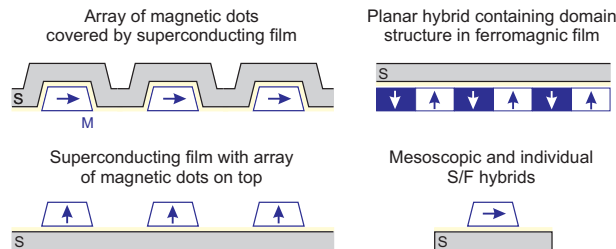
## 1. Introduction

According to the classical Bardeen–Cooper–Schrieffer theory of superconductivity, the ground state of the superconducting condensate consists of electron pairs with opposite spins (the so-called spin-singlet state) bounded via phonon interactions [1, 2]. As early as 1956, Ginzburg [3] pointed out that this fragile state of matter could be destroyed by the formation of a homogeneous ferromagnetic ordering of spins if its corresponding magnetic field exceeds the thermodynamical critical field of the superconductor. Later on, Matthias *et al.* [4, 5, 6] demonstrated that besides the orbital effect (i.e. a pure electromagnetic interaction between the ferromagnetic and superconducting subsystems), there is also a strong suppression of superconductivity arising from the exchange interaction which tends to align the spins of the electrons in detriment of Cooper-pair formation. Anderson and Suhl [7] predicted that a compromise between these antagonistic states can be achieved if the ferromagnetic phase is allowed to break into domains of size much smaller than the superconducting coherence length  $\xi$  in such a way that from the superconductivity point of view, the net magnetic moment averages to zero. Alternatively, Larkin and Ovchinnikov [8] and Fulde and Ferrel [9], theoretically predicted that superconductivity can survive in a uniform ferromagnetic state if the superconducting order parameter is spatially modulated.

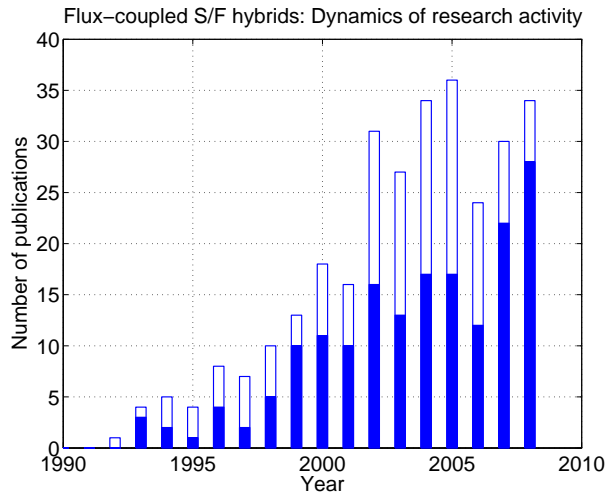
In general terms, the effective polarization of the conduction electrons, either due to the external field  $H_{ext}$  (orbital effect) or the exchange field  $H_{ex}$  (paramagnetic effect), leads to a modification (suppression and modulation) of the superconducting order parameter. Typically, in ferromagnetic metals the exchange field is considerably higher than the internal magnetic field and it dominates the properties of the system. However, in some cases, where both fields can have opposite directions, an effective compensation of the conduction electrons polarization can occur and consequently superconductivity can be recovered at high fields  $H_{ext} \simeq -H_{ex}$  (Jaccarino and Peter [10]). Bulaevskii *et al.* [11] gave an excellent overview of both experimental and theoretical aspects of coexistence of superconductivity and ferromagnetism where both orbital and exchange effects are taken into account.

The progressive development of material deposition techniques and the advent of refined lithographic methods have made it possible to fabricate superconductor-ferromagnet structures (S/F) at nanometer scales. Unlike the investigations dealing with the coexistence of superconductivity and ferromagnetism in ferromagnetic superconductors (for review see Flouquet and Buzdin [12]), the ferromagnetic and superconducting subsystems in the artificial heterostructures can be physically separated. As a consequence, the strong exchange interaction is limited to a certain distance around the S/F interface whereas the weaker electromagnetic interaction can persist to longer distances into each subsystem. In recent reviews, Izyumov *et al.* [13], Buzdin [14] and Bergeret *et al.* [15] discussed in detail the role of proximity effects in S/F heterostructures dominated by exchange interactions†. In order to unveil the effect of electromagnetic coupling it is imperative to suppress proximity effects by introducing an insulating buffer material between the S and F films. In an earlier report, Lyuksyutov and

† In particular, trilayered S/F/S structures with transparent S/F interfaces allow to realize Josephson junctions with an arbitrary phase difference between the superconducting electrodes, which depend on the thickness of the ferromagnetic layer (see, e.g., papers of Prokić *et al.* [16], Ryazanov *et al.* [17], Kontos *et al.* [18], Buzdin and Baladie [19], Oboznov *et al.* [20] and references therein). The antipode F/S/F heterostructures attract a considerable attention in connection with the investigation of unusual properties of such layered hybrid structures governed by the mutual orientation of the vectors of the magnetization in the “top” and “bottom” ferromagnetic layers (see, e.g., papers of Deutscher and Meunier [21], Ledvij *et al.* [22], Buzdin *et al.* [23], Tagirov [24], Baladié *et al.* [25], Gu *et al.* [26, 27], Peña *et al.* [28], Moraru *et al.* [29], Rusanov *et al.* [30], Steiner and Ziemann [31], Singh *et al.* [32]).



**Figure 1.** (color online) Typical examples of considered S/F hybrid systems with dominant orbital interaction.



**Figure 2.** (color online) The histogram shows an increase of the number of publications dealing with the investigations of the S/F hybrids where the conventional low- $T_c$  superconductors interact with magnetic textures mainly via stray magnetic fields: blue bars correspond to the experimental papers, while white bars refer to pure theoretical contributions.

Pokrovsky [33] addressed the physical implications of both electromagnetic coupling and exchange interaction in the S/F systems deep into the superconducting state.

In the present review we are aiming to discuss the thermodynamic and low-frequency transport phenomena in the S/F hybrid structures dominated by electromagnetic interactions. We focus only on the S/F hybrids consisting of conventional low- $T_c$  superconductors without weak links<sup>‡</sup>. The S/F heterostructures with pure electromagnetic coupling can be described phenomenologically using Ginzburg-Landau and London formalisms rather than sophisticated microscopical models. Some typical examples of such structures found in the literature are shown schematically in Fig. 1. As an illustration of the continuous growth of interest in S/F heterostructures with suppressed proximity effect we refer to Fig. 2, which shows the number of the publications during the last two decades.

The review is organized as follows. Section 2 is devoted to the nucleation of the superconducting order parameter under inhomogeneous magnetic fields, induced by single domain walls and periodic domain structures in plain ferromagnetic films or by magnetic dots. A similar problem for individual symmetric microstructures was reviewed by Chibotaru *et al.* [40]. Section 3 is devoted to the static and dynamic properties of S/F systems at low temperatures when the superconducting OP becomes fully developed and the screening effects cannot be disregarded any longer. The vortex pinning properties of the S/F hybrids have been recently analyzed by Vélez *et al.* [41] and the fabrication of ordered magnetic nanostructures has been earlier considered by Martín *et al.* [42]. In the last section 4 we briefly introduce the problem of “soft” magnets in combination with superconducting materials, where now the superconducting currents and the magnetic stray field emanating from the ferromagnetic material mutually influence each other. In the conclusion, we formulate a number of relevant issues that, to our understanding, remain unsettled and deserve further investigations. The appendix summarizes the experimental and theoretical research activities on the considered S/F heterostructures, where we present a classification based on the choice of materials for the experimental research and on the used model for the theoretical treatment.

Importantly, we would like to note already in the Introduction that the literature and references used by the authors in this review by no mean can be considered as a complete set. Due to dynamic and rather complex character of the subject and also to the limited space in this review, inevitably quite a lot of important

<sup>‡</sup> Ferromagnetic dots are shown to induce an additional phase difference in Josephson junctions, leading to a significant modification of the dependence of the Josephson critical current  $I_c$  on the external magnetic field  $H_{ext}$  (so-called Fraunhofer diffraction pattern, see, e.g., textbook of Barone and Paterno [34]), which becomes sensitive to the magnetization of ferromagnetic particle (Aladyshkin *et al.* [35], Vdovichev *et al.* [36], Fraerman *et al.* [37], Held *et al.* [38], Samokhvalov [39]).

and interesting contributions could have been missed and, therefore, in a way, the used references reflect the “working list” of publications the authors of this review are dealing with.

## 2. Nucleation of superconductivity in S/F hybrids (high-temperature limit)

### 2.1. Ginzburg-Landau description of a magnetically coupled S/F hybrid system

#### Derivation of the Ginzburg-Landau equations

In order to describe hybrid structures, consisting of a type-II superconductor and a ferromagnet, for the case that no diffusion of Cooper pairs from superconductor to ferromagnet takes place, the phenomenological Ginzburg-Landau (GL) theory can be used. As a starting point we consider the properties of S/F hybrids for external magnetic fields  $\mathbf{H}_{ext}$  below the coercive field of the ferromagnet which is assumed to be relatively large. In this case the magnetization of the ferromagnet  $\mathbf{M}$  is determined by the magnetic history only and it does neither depend on  $\mathbf{H}_{ext}$  nor on the distribution of the screening currents inside the superconductor. Such a “hard-magnet approximation” is frequently used for a theoretical treatment and it appears to be approximately valid for most part of the experimental studies presented in this section. The review of the properties of hybrid S/F systems consisting of superconductors and soft magnets will be presented later on in section 4.

Following Landau’s idea of phase transitions of the second kind, the equilibrium properties of a system close to the phase transition line can be obtained by minimization of the free energy functional (see, e.g, textbooks of Abrikosov [43], Schmidt [44], Tinkham [45]):

$$G_{sf} = G_{s0} + G_m + \int_V \left\{ \alpha |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 + \frac{1}{4m} \left| -i\hbar \nabla \Psi - \frac{2e}{c} \mathbf{A} \Psi \right|^2 + \frac{\mathbf{B}^2}{8\pi} - \mathbf{B} \cdot \mathbf{M} - \frac{\mathbf{B} \cdot \mathbf{H}_{ext}}{4\pi} \right\} dV, \quad (1)$$

where the integration should be performed over the entire space<sup>†</sup>. Here  $G_{s0}$  is a field- and temperature-independent part of the free energy,  $\alpha = \alpha_0 (T_{c0} - T)$ ,  $\alpha_0$  and  $\beta$  are positive temperature-independent constants,  $\Psi$  is an effective wave function of the Cooper pairs,  $\mathbf{B}(\mathbf{r}) = \text{rot } \mathbf{A}(\mathbf{r})$  is the magnetic field and the corresponding vector potential,  $T_{c0}$  is the critical temperature at  $B = 0$ ,  $e$  and  $m$  are charge and mass of carriers (e.g., electrons), and  $c$  is the speed of light. The term  $G_m$ , which will be explicitly introduced in the last section 4, accounts for the self-energy of the ferromagnet which depends on the particular distribution of magnetization. This term seems to be constant for hard ferromagnets with a fixed distribution of magnetization, therefore it does not influence the OP pattern and the superconducting current distribution in hard S/F hybrids. Introducing a dimensionless wave function  $\psi = \Psi/\Psi_0$ , normalized by the OP value  $\Psi_0 = \sqrt{a_0(T_{c0} - T)/b}$  in saturation, one can rewrite Eq. (1) in the following form

$$G_{sf} = G_{s0} + G_m + \int_V \left\{ \frac{\Phi_0^2}{32\pi^3 \lambda^2} \left( -\frac{1}{\xi^2} |\psi|^2 + \frac{1}{2\xi^2} |\psi|^4 + \left| \nabla \psi + i \frac{2\pi}{\Phi_0} \mathbf{A} \psi \right|^2 \right) + \frac{\mathbf{B}^2}{8\pi} - \mathbf{B} \cdot \mathbf{M} - \frac{\mathbf{B} \cdot \mathbf{H}_{ext}}{4\pi} \right\} dV, \quad (2)$$

expressed via the temperature-dependent coherence length  $\xi^2 = \hbar/[4m a_0(T_{c0} - T)]$ , the London penetration depth  $\lambda^2 = mc^2 b/[8\pi e^2 a_0(T_{c0} - T)]$ , and the magnetic flux quantum  $\Phi_0 = \pi \hbar c/|e|$ .

Although the Ginzburg-Landau model was proven to be consistent only at temperatures close to the superconducting critical temperature (Gorkov [46]), the applicability of this model seems to be much broader, at least from a qualitative point of view. After minimization of the free energy functional Eq. (2) with respect to the order parameter (OP) wave function  $\psi$  and  $\mathbf{A}$  respectively, one can derive the two coupled Ginzburg-Landau equations [43, 44, 45]:

$$-\xi^2 \left( \nabla + i \frac{2\pi}{\Phi_0} \mathbf{A} \right)^2 \psi - \psi + |\psi|^2 \psi = 0, \quad (3)$$

$$\text{rot rot } \mathbf{A} = \frac{4\pi}{c} \mathbf{j}_s + 4\pi \text{rot } \mathbf{M} + \frac{4\pi}{c} \mathbf{j}_{ext}, \quad (4)$$

<sup>†</sup> Hereafter we used the Gauss (centimeter-gram-second) system of units, therefore all vectors  $\mathbf{B}$ ,  $\mathbf{M}$ , and  $\mathbf{H}_{ext}$  have the same dimensionality:  $[B]=\text{Gauss (G)}$ ,  $[M]=\text{Oersted (Oe)}$ ,  $[H_{ext}]=\text{Oersted (Oe)}$ .



where

$$\mathbf{j}_s = \frac{c}{4\pi} \frac{|\psi|^2}{\lambda^2} \left( \frac{\Phi_0}{2\pi} \nabla \Theta - \mathbf{A} \right)$$

represents the density of superconducting currents, while  $\mathbf{j}_{ext} = (c/4\pi) \text{rot} \mathbf{H}_{ext}$  is the density of the currents corresponding to external sources, and  $\Theta$  is the OP phase,  $\psi(\mathbf{r}) = f(\mathbf{r}) e^{i\Theta(\mathbf{r})}$ .

#### Linearized GL equation

It is quite natural to expect that at the initial stage of the formation of superconductivity [i.e. close to the phase transition line  $T_c(H_{ext})$ , which separates the normal and superconducting state in the  $T - H_{ext}$  plane], the density of the superconducting condensate will be much smaller than the fully developed OP value:  $|\psi|^2 \ll 1$ . This allows one to neglect: (i) the nonlinear term  $|\psi|^2 \psi$  in Eq. (3) and, (ii) the corrections to the vector potential  $\mathbf{A}$  caused by the screening currents in Eq. (4), since the supercurrents  $\mathbf{j}_s$  are also proportional to  $|\psi|^2$ . Thus, the nucleation of superconductivity can be analyzed in the framework of the linearized GL equation [43, 44, 45]:

$$-\left(\nabla + i \frac{2\pi}{\Phi_0} \mathbf{A}\right)^2 \psi = \frac{1}{\xi^2} \psi, \quad (5)$$

in a given magnetic field described by the vector potential distribution

$$\mathbf{A} = \frac{1}{c} \int \frac{\mathbf{j}_{ext}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' + \int \frac{\text{rot} \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}', \quad (6)$$

The solution of Eq. (5) consists of a set of eigenvalues  $(1/\xi^2)_n$ , corresponding to the appearance of certain OP pattern  $\psi_n$ , for every value of the applied magnetic field  $H_{ext}$ . The critical temperature of the superconducting transition  $T_c$  is determined by the lowest eigenvalue of the problem:  $T_c = T_{c0} \{1 - \xi_0^2 (1/\xi^2)_{min}\}$ .

#### The phase boundary for plain superconducting films

First, we would like to present the well-known solution of the linearized GL equation Eq. (5) corresponding to the OP nucleation in a plain superconducting film, infinite in the lateral direction and placed in a transverse *uniform* magnetic field  $\mathbf{H}_{ext} = H_{ext} \mathbf{z}_0$  [43, 44, 45]. Taking the gauge  $A_y = xH_{ext}$ , one can see that Eq. (5) depends explicitly on the  $x$ -coordinate only, therefore its general solution can be written in the form:  $\psi = f(x) e^{iky+iqz}$ , where the wave vectors  $k$  and  $q$  should adjust themselves to provide the maximization of the  $T_c$  value. Using this representation in Eq. (5), it is easy to see that the spectrum of eigenvalues  $(1/\xi^2)_n$  is similar to the energy spectrum of the harmonic oscillator but shifted:  $(1/\xi^2)_n = 2\pi(2n+1)H_{ext}/\Phi_0 + q^2$  and the  $(1/\xi^2)$  minimum (the maximum of  $T_c$ ) corresponds to  $n=0$  and  $q=0$  for any  $H_{ext}$  value,  $(1/\xi^2)_{min} = 2\pi H_{ext}/\Phi_0$ . The critical temperature of the superconducting transition<sup>†</sup> as a function of a uniform transverse magnetic field is given by  $T_c = T_{c0} [1 - 2\pi\xi_0^2 H_{ext}/\Phi_0]$ , or

$$1 - \frac{T_c}{T_{c0}} = \frac{|H_{ext}|}{H_{c2}^{(0)}}, \quad (7)$$

where  $H_{c2}^{(0)} = \Phi_0/(2\pi\xi_0^2)$  is the upper critical field at  $T=0$ . The inversely proportional dependence of the shift of the critical temperature  $1 - T_c/T_{c0}$  on the square of the OP width  $\ell_H^2$  can be interpreted in terms of the quantum-size effect for Cooper pairs in a uniform magnetic field. It should be mentioned that the effect of sample's topology on the eigenenergy spectrum  $(1/\xi^2)_n$  becomes extremely important for mesoscopic superconducting systems, whose lateral dimensions are comparable with the coherence length  $\xi$ . Indeed, this additional confinement of the OP wave function significantly modifies the OP nucleation in mesoscopic superconductors and the corresponding phase boundaries  $T_c(H_{ext})$  differ considerably from that typical for bulk samples and films infinite in the lateral directions (see Chibotaru *et al.* [40], Moshchalkov *et al.* [47, 48], Berger and Rubinstein [49]).

<sup>†</sup> It is well known that superconductivity nucleates in the form of a Gaussian-like OP wave function  $\psi(x, y) = e^{-(x-x_0)^2/2\ell_H^2} e^{iky}$ , localized in the lateral direction at distances of the order of the so-called magnetic length  $\ell_H^2 = \Phi_0/(2\pi|H_{ext}|)$  and uniform over the film thickness. The oscillatory factor  $e^{iky}$  describes the displacement of the OP maximum positioned at  $x_0 = k\Phi_0/(2\pi H_{ext})$  without a change of the  $(1/\xi^2)_{min}$  value. It is interesting to note that the confinement of the OP wave function is determined by the magnetic length  $\ell_H$ , i.e. the OP width is a function of the external field  $H_{ext}$ . On the other hand, the temperature-dependent coherence length  $\xi$  is a natural length scale describing the spatial OP variations. The equality  $\ell_H^2 = \xi^2$  defines the same phase boundary in the  $T - H_{ext}$  plane as that given by Eq. (7).

## 2.2. Magnetic confinement of the OP wave function in an inhomogeneous magnetic field: general considerations

The main focus in this section is to describe the nucleation of the superconducting order parameter in a static *non-uniform* magnetic field  $\mathbf{H}_{ext} + \mathbf{b}(\mathbf{r})$  based on a simple approach<sup>†</sup>. This method makes it possible to see directly a correspondence between the position of the maximum of the localized wave function  $\psi$  and the critical temperature  $T_c$  in the presence of spatially-modulated magnetic field  $\mathbf{b}(\mathbf{r})$ , generated by ferromagnet. For simplicity, we assume that the thin superconducting film is infinite in the  $(x, y)$ -plane, i.e. perpendicular to the direction of the external field  $\mathbf{H}_{ext} = H_{ext} \mathbf{z}_0$ . This allows us to neglect the possible appearance of superconductivity in the sample perimeter (surface superconductivity [43, 44, 45]) and focus only on the effect arising from the nonuniform magnetic field.

### Importance of out-of-plane component of the field

It should be emphasized that the formation (or destruction) of superconductivity in thin superconducting films is sensitive to the spatial variation of the out-of-plane component of the total magnetic field. Indeed, the upper critical fields  $H_{c2}^\perp$  and  $H_{c2}^\parallel$  for the out-of-plane and in-plane orientation for a uniform applied magnetic field can be estimated as follows [43, 44, 45]:

$$H_{c2}^\perp \sim \frac{\Phi_0}{\xi^2}, \quad H_{c2}^\parallel \sim \frac{\Phi_0}{\xi D_s}, \quad (8)$$

where  $D_s$  is the thickness of the superconducting sample. For rather thin superconducting films and/or close to the superconducting critical temperature  $D_s \ll \xi = \xi_0(1 - T/T_{c0})^{-1/2}$ , therefore  $H_{c2}^\perp \ll H_{c2}^\parallel$ . In other words, superconductivity will generally be destroyed by the out-of-plane component of the magnetic field rather than by the in-plane component, and thus, to a large extent, the spatial distribution of the out-of-plane component determines the OP nucleation in thin-film structures. Since a uniform magnetic field is known to suppress the critical temperature, one can expect that the highest  $T_c$  value should correspond to the OP wave function localized near regions with the lowest values of the perpendicular magnetic field  $|B_z(\mathbf{r})|$  provided that  $B_z = H_{ext} + b_z(\mathbf{r})$  varies slowly in space.

If the field  $H_{ext}$  exceeds the amplitude of the internal field modulation (i.e.  $H_{ext} < -\max b_z$  and  $H_{ext} > -\min b_z$ ), the total magnetic field is non-zero in the whole sample volume, and the favorable positions for the OP nucleation are at the locations of minima of  $|B_z(\mathbf{r})| = |H_{ext} + b_z(\mathbf{r})|$ . If the characteristic width  $\ell_\psi$  of the OP wave function, which will be defined later, is much less than the typical length scale  $\ell_b$  of the magnetic field variation, then locally the magnetic field can be considered as uniform at distances of the order of  $\ell_\psi$  and it approximately equals to  $\min |H_{ext} + b_z(\mathbf{r})|$ . Then, using the standard expression for the upper critical field Eq. (7) and substituting the effective magnetic field instead of the applied field, one can obtain the following estimate for the phase boundary

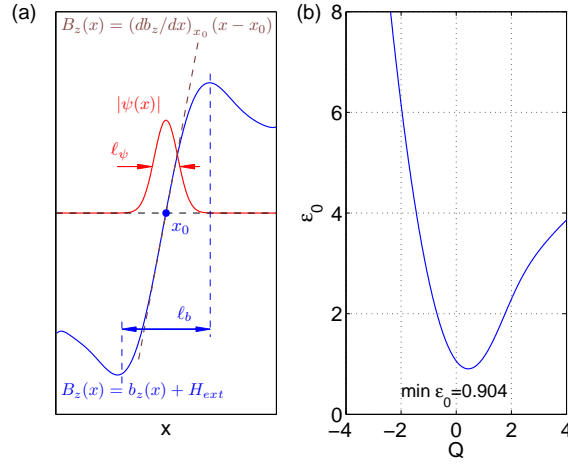
$$1 - \frac{T_c}{T_{c0}} \simeq \min \frac{|H_{ext} + b_z(\mathbf{r})|}{H_{c2}^{(0)}}, \quad |H_{ext}| \gg \max |b_z|. \quad (9)$$

According to this expression, the dependence  $T_c(H_{ext})$  is still linear asymptotically even in the presence of a nonuniform magnetic field. However, the critical field will be shifted upwards (for  $H_{ext} > 0$ ) and downward (for  $H_{ext} < 0$ ) by an amount close to the amplitude of the field modulation. Generally speaking, such a “magnetic bias” can be asymmetric with respect to the  $H_{ext} = 0$  provided that  $\max b_z(x) \neq |\min b_z(x)|$ .

For relatively low  $H_{ext}$  values, when the absolute value of the external field is less than the amplitude of the field modulation ( $-\max b_z < H_{ext} < -\min b_z$ ), the  $z$ -component of the total magnetic field  $H_{ext} + b_z(x)$  becomes zero locally somewhere inside the superconducting film. As a result, superconductivity is expected to appear first near the positions where  $H_{ext} + b_z(\mathbf{r}_0) = 0$  [see panel (a) in Fig. 3]. It is natural to expect that the details of the OP nucleation depend strongly on the exact topology of the stray field as well as the field gradient near the lines of zero field. As an example we will analyze the formation of superconductivity in a thin superconducting film placed in an inhomogeneous magnetic field modulated along certain direction.

### OP nucleation in a magnetic field modulated in one direction

<sup>†</sup> We introduce the following notations:  $\mathbf{b} = \text{rot } \mathbf{a}$  characterizes the non-uniform component of the magnetic field only, while  $\mathbf{B} = \mathbf{H}_{ext} + \mathbf{b} = \text{rot } \mathbf{A}$  is the total magnetic field distribution, the external field  $\mathbf{H}_{ext}$  is assumed to be uniform.



**Figure 3.** (color online) (a) Schematic representation of the OP wave function  $\psi(x)$  localized near the point  $x_0$ , where the  $z$ -component of the total magnetic field  $B_z = H_{ext} + b_z$  vanishes. Provided that the OP width is much smaller than the typical length scales of the magnetic field ( $\ell_\psi \ll \ell_b$ ), the actual field distribution  $B_z(x)$  can be approximated by a linear dependence  $B_z(x) \simeq (db_z/dx)_{x_0}(x - x_0)$ . (b). Energy spectrum  $\varepsilon_0$  vs.  $Q$  of the model problem Eq. (11), after Aladyshkin *et al.* [51].

Following Aladyshkin *et al.* [50], we estimated the dependence of  $T_c(H_{ext})$  for a thin superconducting film in the presence of a non-uniform magnetic field modulated along the  $x$ -direction, where  $(x, y, z)$  is the Cartesian reference system. Let the external field be oriented perpendicular to the plane of the superconducting film,  $\mathbf{H}_{ext} = H_{ext} \mathbf{z}_0$ , while the  $z$ -component of the total magnetic field vanishes at the point  $x_0$ , i.e.  $H_{ext} + b_z(x_0) = 0$ . The vector potential, corresponding to the field distribution  $B_z(x) = H_{ext} + b_z(x)$ , can be chosen in the form  $A_y(x) = xH_{ext} + a_y(x)$ , where  $b_z = da_y/dx$ . Since there is no explicit dependence on the  $y$ -coordinate in the linearized GL equation Eq. (5), the solution uniform over the sample thickness can be generally found as  $\psi(x, y) = f_k(x) e^{iky}$  and the absolute value of the OP satisfies the following equation:

$$-\frac{d^2 f_k}{dx^2} + \left( \frac{2\pi}{\Phi_0} x H_{ext} + \frac{2\pi}{\Phi_0} a_y(x) - k \right)^2 f_k = \frac{1}{\xi^2} f_k. \quad (10)$$

Now the parameter  $k$  cannot be excluded by a shift of the origin of the reference system, therefore one should determine the particular  $k$  value in order to minimize  $(1/\xi_0^2)(1 - T_c/T_{c0})$  and thus, to maximize the  $T_c$  value.

For an unidirectional modulation of the field, the curves of zero field, where we expect the preferable OP nucleation, are straight lines parallel to the  $y$ -axis, and their positions depend on the external field,  $x_0 = x_0(H_{ext})$ . Expanding the vector potential inside the superconducting film in a power series around the point  $x_0$ , one can get

$$A_y(x) \simeq x_0 H_{ext} + a_y(x_0) + \frac{1}{2} b'_z(x_0)(x - x_0)^2 + \dots$$

This local approximation is valid as long as  $|b''_z(x_0)\ell_\psi/b'_z(x_0)| \ll 1$ . Introducing a new coordinate  $\tau = (x - x_0)/\ell_\psi$  and the following auxiliary parameters  $\ell_\psi$  and  $Q_k$

$$\ell_\psi = \sqrt[3]{\frac{\Phi_0}{\pi|b'_z(x_0)|}}, \quad Q_k = -\sqrt[3]{\frac{\Phi_0}{\pi b'_z(x_0)}} \left( \frac{2\pi}{\Phi_0} x_0 H_{ext} + \frac{2\pi}{\Phi_0} a_y(x_0) - k \right),$$

we can reduce Eq. (10) to the bi-quadratic dimensionless equation

$$-\frac{d^2 f}{d\tau^2} + (\tau^2 - Q_k)^2 f = \varepsilon f, \quad \text{where} \quad \varepsilon = \frac{\ell_\psi^2}{\xi_0^2} \left( 1 - \frac{T}{T_{c0}} \right). \quad (11)$$

Thus, the problem of the calculation of the highest  $T_c$  value in the presence of an arbitrary slowly-varying magnetic field as a function of both the external field and the parameters of the “internal” fields is reduced to the determination of the lowest eigenvalue  $\varepsilon_0 = \varepsilon_0(Q)$  of the model equation Eq. (11). As was shown in Ref. [51], the function  $\varepsilon_0(Q)$  is characterized by the following asymptotical behavior:  $\varepsilon_0(Q) \simeq Q^2 + \sqrt{-Q}$  for  $Q \ll -1$  and  $\varepsilon_0(Q) \simeq 2\sqrt{Q}$  for  $Q \gg 1$  and it has the minimum value  $\varepsilon_{min} = 0.904$  [the panel (b) in Fig. 3].

Extracting  $T_c$  from  $\varepsilon_0 = (1 - T_c/T_{c0}) \cdot \ell_\psi^2/\xi_0^2$ , the approximate expression of the phase boundary takes the following form:

$$1 - \frac{T_c}{T_{c0}} \simeq \frac{\xi_0^2}{\ell_\psi^2} \min_k \varepsilon_0(Q_k) \simeq \xi_0^2 \left( \pi \frac{|b'_z(x_0)|}{\Phi_0} \right)^{2/3}, \quad -\max b_z < H_{ext} < -\min b_z. \quad (12)$$

If there are several points  $x_{0,i}$  where the external field compensates the field generated by the ferromagnetic structure, then the right-hand part of Eq. (12) should be minimized with respect to  $x_{0,i}$ . The application of Eq.(12) for the model cases  $b_z = 4M_s \arctan(D_f/x)$  (single domain wall) and  $b_z = B_0 \sin(2\pi x/w)$  (periodic domain structure) were given in Ref. [50].

#### *OP nucleation in axially-symmetrical magnetic field*

Similar to the discussion above, one can expect that in the presence of an axially-symmetrical magnetic field superconductivity will nucleate in the form of ring-shaped channels of radius  $r = r_0$ , where  $H_{ext} + b_z(r_0) = 0$ . The independence of the linearized GL equation Eq. (5) on the angular  $\varphi$ -coordinate results in a conservation of the angular momentum (vorticity)  $L$  of the superconducting wave function. Thus, a nonuniform magnetic field makes it possible to have an appearance of giant (multiquanta) vortex states, which are energetically unfavorable in plain (non-perforated) large-area superconducting films, but have been observed in mesoscopic superconductors (Moshchalkov *et al.* [47, 48], Berger and Rubinstein [49]) and nanostructured films with antidot lattices (Baert *et al.* [52], Moshchalkov *et al.* [53]). Expanding the vector potential in the vicinity of  $r_0$  and repeating similar transformations as above, one can get the following approximate expression† for the phase transition line (Aladyshkin *et al.* [51, 54])

$$1 - \frac{T_c}{T_{c0}} \simeq \frac{\xi_0^2}{\ell_\psi^2} \left( \min_L \varepsilon_0(Q_L) - \frac{\ell_\psi^2}{4r_0^2} \right), \quad -\max b_z < H_{ext} < -\min b_z,$$

where the parameters  $\ell_\psi = \sqrt{\Phi_0/(\pi |db_z/dr|_{r_0})}$  and  $Q_L = -[2\pi r_0 A_\varphi(r_0)/\Phi_0 - L] \ell_\psi/r_0$  depend on the external field. Thus, this model predicts field-induced transitions between giant vortex states with different vorticities. Since the vorticity  $L$  is a discrete parameter, the changes of the favorable  $L$  value while sweeping the external field leads to abrupt changes in  $dT_c/dH_{ext}$ . Similar periodic oscillations of  $T_c$  were originally observed by Little and Parks [55, 56] for a superconducting cylinder in a parallel magnetic field and later for any mesoscopic superconductor in a perpendicular magnetic field (for a review see Chibotaru *et al.* [40]).

#### *Effect of nonuniform magnetic field on two-dimensional electron gas*

It is interesting to note that there is a formal similarity between the linearized GL equation (5) and the stationary Schrödinger equation (see, e.g., Ref. [57]) for a charged free spinless particle in a magnetic field

$$-\frac{\hbar^2}{2m} \left( \nabla - \frac{ie}{\hbar c} \mathbf{A} \right)^2 \psi = E\psi, \quad (13)$$

where  $\psi$  is the single particle wave function,  $e$  is the charge and  $m$  is the mass of this particle. Based on this analogy, one can map the results, obtained for a normal electronic gas in the ground state, on the properties of the superconducting condensate near the “superconductor–normal metal” transition on the  $T - H_{ext}$  diagram.

In particular, the effect of a unidirectional magnetic field modulation on the energy spectrum of a two-dimensional electronic gas (2DEG) was analyzed by Müller [58], Xue and Xiao [59], Peeters and Vasilopoulos [60], Peeters and Matulis [61], Wu and Ulloa [62], Matulis *et al.* [63], Ibrahim and Peeters [64], Peeters *et al.* [65], Gumbs and Zhang [66], Reijniers and Peeters [67, 68], Nogaret *et al.* [69, 70]. Remarkably, in a magnetic field varying linearly along certain direction, quasi-classical electronic trajectories propagating perpendicularly to the field gradient  $\nabla B_z$  are confined to a narrow one-dimensional channel localized around the region where  $B_z = 0$  [58]. A more detailed numerical treatment revealed a lifting of the well-known degeneracy of the Landau states on the centers of the Larmor’s orbit, inherent to electrons in a uniform magnetic field. Periodic magnetic field patterns with zero and nonzero average value was shown to transform the standard Landau spectrum  $E = \hbar\omega_c(n + 1/2)$  into a periodic  $E(k_y)$  dependence, describing the broadening of the discrete Landau levels

† Note that the case of a point magnetic dipole approximation fails for  $L = 0$  since the maximum of the OP wave function is located at  $r = 0$  and this theory cannot correctly describe neither the OP nucleation nor the phase boundary  $T_c(H_{ext})$  at negative  $H_{ext}$  values close to the compensation field  $B_0 = -\max b_z(r)$ .

into minibands as in the case of one-dimensional potential (here  $\omega_c = |e|H_{ext}/(mc)$  is the cyclotron frequency). An oscillatory change of the width of energy bands as  $H$  is swept was shown to give rise to oscillations in the magnetoresistance of 2DEG at low  $H_{ext}$  values, which reflect the commensurability between the diameter of cyclotron orbit at the Fermi level and the period of the magnetic field modulation [59]. The mentioned oscillatory magnetoresistance due to commensurability effects was later on corroborated experimentally by Carmona *et al.* [71]. The influence of two-dimensional magnetic modulations on the single-particle energy spectrum was theoretically considered by Hofstadter [72]. A modification of the scattering of two-dimension electrons due to the presence of either ferromagnetic dots or superconducting vortices was shown to lead to a nontrivial change of the conductivity in various hybrid systems: 2DEG/superconductor (Geim *et al.* [73], Brey and Fertig [74], Nielsen and Hedegard [75], Reijniers *et al.* [76]) and 2DEG/ferromagnet (Khveshchenko and Meshkov [77], Ye *et al.* [78], Solimany and Kramer [79], Ibrahim *et al.* [80], Sim *et al.* [81], Dubonos *et al.* [82], Reijniers *et al.* [83, 84]). The effect of inhomogeneous magnetic field on the weak-localization corrections to the classical conductivity of disordered 2DEG was considered by Rammer and Shelankov [85], Bending [86], Bending *et al.* [87, 88, 89], Mancoff *et al.* [90], Shelankov [91], Wang [92].

### 2.3. Planar S/F hybrids with ferromagnetic bubble domains: theory

The aforementioned Ginzburg-Landau formalism can be applied in the case of a non-uniform magnetic field generated by the domain structure of a plain magnetic film. The problem of the OP nucleation in planar S/F hybrid structures was theoretically analyzed for hard ferromagnets characterized by an out-of-plane magnetization  $\mathbf{M} = M_z(x) \mathbf{z}_0$  by Aladyshkin *et al.* [50], Buzdin and Melnikov [93], Samokhin and Shirokoff [94], Aladyshkin *et al.* [95], Gillijns *et al.* [96]. It is also worth mentioning the pioneer paper of Pannetier *et al.* [97], where the OP nucleation in a periodic sinusoidal magnetic field generated by a meander-like lithographically-prepared metallic coil was considered.

The distribution of the vector potential  $\mathbf{a}(\mathbf{r})$  can be obtained, either by integration of the last term in the r.h.s. of Eq. (6) or by a direct consideration of the magnetostatic problem. Provided that the width of the domain walls  $\delta$  is much smaller than other relevant length scales, the field distributions can be calculated analytically for some simple configurations (Aladyshkin *et al.* [95], Sonin [98, 99]). Choosing the gauge  $A_y = x H_{ext} + a_y(x, z)$ , one can easily see that the linearized GL equation Eq. (5) does not depend on the  $y$ -coordinate, hence we can generally find the solution in the form  $\psi(x, y, z) = f_k(x, z) \exp(iky)$ , where the function  $f_k(x, z)$  should be determined from the following 2D equation:

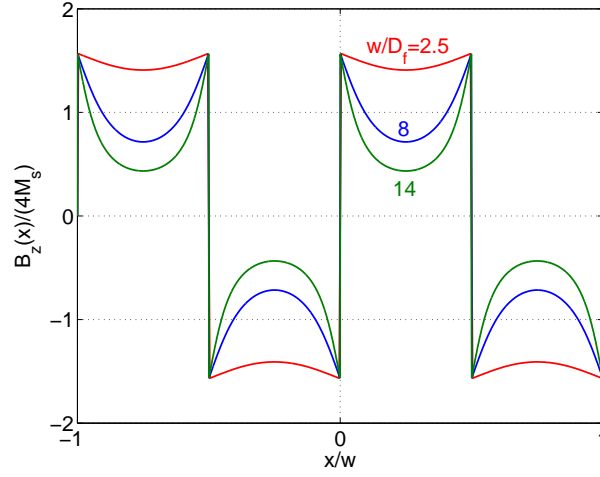
$$-\frac{\partial^2 f_k}{\partial x^2} - \frac{\partial^2 f_k}{\partial z^2} + \left[ \frac{2\pi}{\Phi_0} a_y(x, z) + \frac{2\pi}{\Phi_0} x H_{ext} - k \right]^2 f_k = \frac{1}{\xi^2} f_k, \quad (14)$$

If the superconducting film has insulating interfaces at the top and bottom surfaces and  $\mathbf{A} \cdot \mathbf{n} = 0$  at the surface  $\partial V_s$  of the superconductor, one should apply the standard boundary conditions:  $\partial f_k / \partial n \Big|_{\partial V_s} = 0$  (here  $\mathbf{n}$  is the normal vector).

The spatial distribution of the magnetic field, induced by the periodic 1D domain structure, strongly depends on the relationship between the width of the magnetic domains  $w$  and the thickness of the ferromagnetic film  $D_f$  as well as on the distance  $h$  between superconductor and ferromagnet (Fig. 4). If the superconducting film thickness  $D_s$  is much smaller than the typical length scales of the nonuniform magnetic field ( $w$  and  $D_f$ ), in a first approximation one can neglect the OP variations in the  $z$ -direction and omit the term  $\partial^2 f_k / \partial z^2$  in Eq. (14). As a result, the OP nucleation in a thin superconducting film is determined by the spatial profile of the perpendicular magnetic field only, while the effect of the parallel field can be ignored.

#### *Criterion for the development of domain-wall superconductivity*

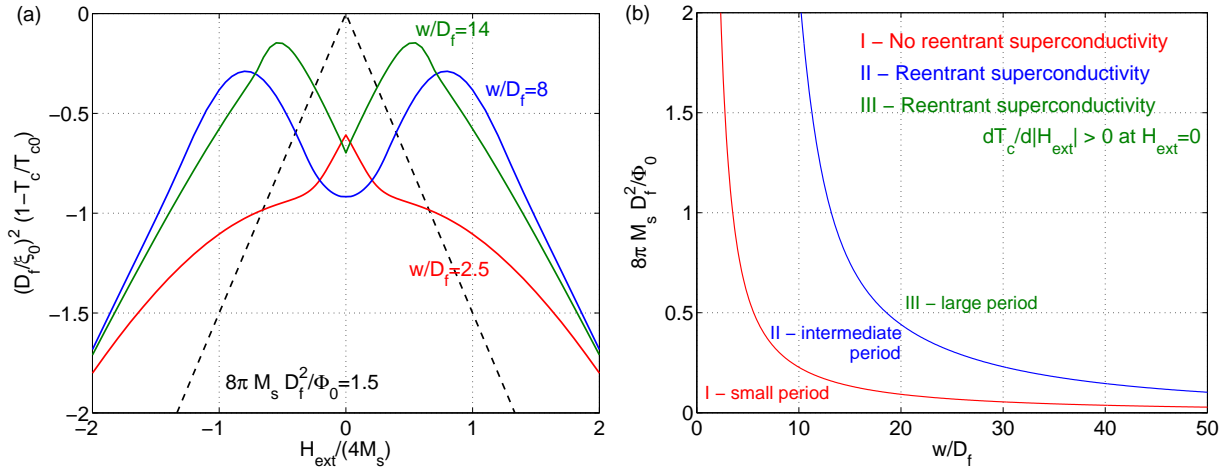
In this section we will discuss the possibility of localizing the OP wave function near a domain wall at zero external field. Obviously, the regime of domain-wall superconductivity (DWS) can be achieved only if the typical  $b_z^*$  value inside the magnetic domains is rather large in order to provide an exponential decay of the OP, described by the effective magnetic length  $\ell_b^* = \sqrt{\Phi_0 / 2\pi |b_z^*|}$ , within a half-width of the domain:  $\ell_b^* < w/2$ . For thick ferromagnetic films ( $w/D_f \ll 1$ ) the magnetic field inside domains is almost uniform and it can be estimated as  $b_z^* \simeq 2\pi M_s$ , giving us the rough criterion of the realization of the DWS regime and the critical



**Figure 4.** (Color online) Transverse  $z$ -component of the magnetic field, induced by one-dimensional periodic distribution of magnetization with the amplitude  $M_s$  and the period  $w$ , calculated at the distance  $h \ll D_f$  above the ferromagnetic film of a thickness  $D_f$ , adapted from Aladyshkin and Moshchalkov [95].

temperature  $T_c^{(0)}$  at  $H_{ext} = 0$ :  $\pi^2 M_s w^2 / \Phi_0 > 1$  and  $(T_{c0} - T_c^{(0)}) / T_{c0} \simeq 2\pi M_s / H_{c2}^{(0)}$ . Of course,  $2\pi M_s / H_{c2}^{(0)}$  should be less than unity otherwise superconductivity will be totally suppressed.

By applying an external field on the order of the compensation field,  $H_{ext} \simeq 2\pi M_s$ , one can get local compensation of the field above the domains with opposite polarity and a doubling of the field above the domains of the same polarity. Since superconductivity is expected to form at regions with zero field (which are  $w$  wide), the maximal critical temperature can be estimated as follows:  $(T_{c0} - T_c^{max}) / T_{c0} \simeq \xi_0^2 / w^2$  (a consequence of the quantum-size effect for Cooper pairs in nonuniform magnetic field). Therefore,  $T_c^{max}$  will



**Figure 5.** (Color online) (a) Examples of the phase transition lines  $T_c(H_{ext})$  for a planar S/F structure containing a periodic 1D domain structure ( $M_s$  is the saturated magnetization,  $D_f$  is the ferromagnetic film thickness,  $w$  is the period of the domain structure, the thickness of the superconducting film  $D_s \ll (D_f, w)$  and the separation between superconducting and ferromagnetic films  $h/D_f \ll 1$ ), adapted from Aladyshkin and Moshchalkov [95]. Black dashed line correspond to the  $T_c(H_{ext})$  dependence in the absence of the non-uniform field.

(b) Different regimes of localized superconductivity in the presence of a 1D domain structure in the  $M_s - w$  plane, obtained numerically for  $D_s \ll (D_f, w)$  and  $h/D_f \ll 1$ , adapted from Aladyshkin and Moshchalkov [95]. In regions II and III the phase boundary  $T_c(H_{ext})$  exhibits reentrant superconductivity. The slope  $dT_c/d|H_{ext}|$  at  $H_{ext} = 0$  can be positive (III), zero (II) or negative (II, near the separating line I-II). Region I corresponds to the monotonic  $T_c(H_{ext})$  dependence.

exceed  $T_c^{(0)}$ , pointing out to the non-monotonous  $T_c(H_{ext})$  dependence for the same  $M_s$  and  $w$  parameters, which are necessary to have the DWS regime at  $H_{ext} = 0$ . The typical phase boundary  $T_c(H_{ext})$ , corresponding to the DWS regime at  $H_{ext} = 0$  and shown in panel (a) in Fig. 5, is characterized by the presence of a pronounced reentrant behavior and the parabolic dependence of  $T_c$  on  $H_{ext}$  at low fields (curve labelled  $w/D_f = 8$ ). This type of phase boundary was predicted by Pannetier *et al.* [97] for a superconducting film in a field of parallel metallic wires carrying a dc current, and by Buzdin and Melnikov [93] and Aladyshkin *et al.* [50, 95] for planar S/F hybrids.

#### *Localized superconductivity in S/F hybrids for $w/D_f \ll 1$ and $w/D_f \gg 1$*

For S/F hybrids with smaller periods of the field modulation ( $\pi^2 M_s w^2 / \Phi_0 \ll 1$ ) the OP distribution cannot follow the rapid field variations and, as a consequence, at  $H_{ext} \simeq 0$  there is a broad OP wave function, spreading over several domains and resulting in an effective averaging of the nonuniform magnetic field. In this case the critical temperature was shown to decrease monotonically with increasing  $|H_{ext}|$  [curve labelled  $w/D_f = 2.5$  in Fig. 5(a)], similar to the case of superconducting films in a uniform magnetic field. By applying an external field, one can shrink the width of the OP wave function and localize it within one half-period above the domains with opposite magnetization. The interplay between both, the external field and the periodic magnetic field, which determines the resulting OP width, leads to a sign change of the second derivative  $d^2 T_c / dH_{ext}^2$ . At high  $H_{ext}$  values the width of the OP wave function, positioned at the center of the magnetic domain, is determined by the local field  $B_{loc} \simeq |H_{ext}| - 2\pi M_s$ , therefore we come to a biased linear dependence  $1 - T_c/T_{c0} \simeq ||H_{ext}| - 2\pi M_s|/H_{c2}^{(0)}$ . These qualitative arguments were supported by numerical solutions of the linearized GL equation [95].

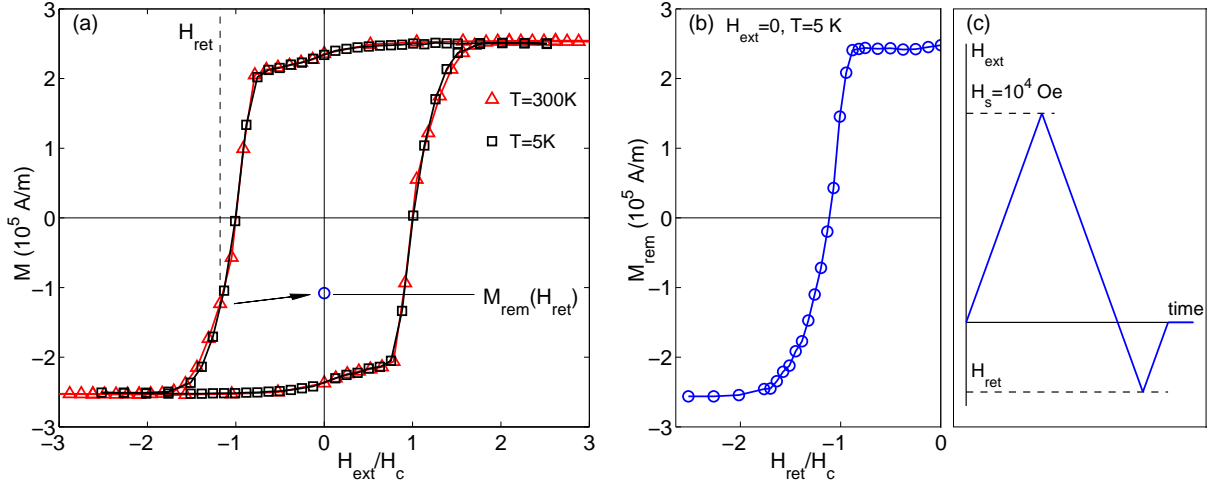
The case  $w/D_f \gg 1$  should be treated separately since the  $z$ -component of the field inside the magnetic domains is very inhomogeneous: the absolute value  $|b_z(x)|$  reaches a minimum  $b_z^* = 8\pi M_s D_f / w$  at the domain center, while the maximal value is still equal to  $2\pi M_s$  at the domain walls. It was shown that the  $|b_z(x)|$  minima are favorable for the OP nucleation at  $H_{ext} = 0$ . In this regime, the OP localization in the center of the domains at  $H_{ext} = 0$  is possible as long as  $2\pi^2 M_s w D_f / \Phi_0 > 1$ . At the same time the nucleation near domain walls is suppressed by the mentioned field enhancement near the domain walls. The sudden displacement of the localized OP wave function between the centers of the domains of positive and negative magnetization, when inverting the  $H_{ext}$  polarity, results in a new type of phase boundary  $T_c(H_{ext})$  with a singularity at  $H_{ext} = 0$  [95]. It is important to note that, for  $w/D_f \gg 1$  and  $H_{ext} = 0$  the critical temperature *increases* linearly with almost the same slope  $dT_c/d|H_{ext}| = T_{c0}/H_{c2}^{(0)}$  as the  $T_c$  value decreases in an applied uniform magnetic field [curve labelled  $w/D_f = 14$  in Fig. 5 (a)].

#### *2.4. Planar S/F hybrids with ferromagnetic bubble domains: experiments*

##### *OP nucleation in perpendicular magnetic field*

To the best of our knowledge, the first observation of reentrant superconductivity<sup>†</sup> in planar S/F hybrids was reported by Yang *et al.* [101] who measured the electrical resistance of a superconducting Nb film grown on top of a ferromagnetic BaFe<sub>12</sub>O<sub>19</sub> substrate characterized by an out-of-plane magnetization. Later on, the same system Nb/BaFe<sub>12</sub>O<sub>19</sub> was examined by Yang *et al.* in Ref. [102]. From the parameters typical for the domain structure in BaFe<sub>12</sub>O<sub>19</sub> single crystals and Nb films ( $M_s \simeq 10^2$  Oe,  $w \simeq 2$   $\mu$ m,  $D_f \simeq 90$   $\mu$ m,  $H_{c2}^{(0)} \simeq 30$  kOe), the following estimates can be obtained:  $w/D_f \simeq 0.02$ ,  $\pi^2 M_s w^2 / \Phi_0 > 10^2$  and  $2\pi M_s / H_{c2}^{(0)} \simeq 0.02$ . Therefore, such a ferromagnet is suitable for the realization of the DWS regime at  $H_{ext} = 0$ . The appearance of these localized superconducting paths guided by domain walls was shown to result in a broadening of the superconducting resistive transition at low magnetic fields. As the field  $H_{ext}$  is ramped up, the superconducting areas shift away from the domain walls towards the wider regions above the domains with an opposite polarity (so-called reversed-domain superconductivity, RDS) where the absolute value of the total magnetic field is minimal because of the

<sup>†</sup> The experimental observation of the influence of a periodic magnetic field, generated by an array of parallel wires with current  $I$  flowing alternatively in opposite directions, on the properties of an Al superconducting bridge was reported by Pannetier *et al.* [97]. Since the  $\max|b_z| \propto I$ , reentrant superconductivity can be realized for rather high  $I$  values, as it was shown experimentally. It should be noted that already in sixties Artley *et al.* [100] experimentally studied the effect of the domain walls in a thin permalloy film on the superconducting transition of a thin indium film.



**Figure 6.** (color online) Preparation of the magnetic state in a ferromagnetic Co/Pt film with a desirable remanent magnetization  $M_{rem}$ .

(a) Magnetization loops  $M(H_{ext})$  at 300 K (triangles) and 5 K (squares), the magnetic field axis is normalized by the corresponding coercive fields  $H_c$ , adapted from Gillijns *et al.* [96];  
 (b) Remanent magnetization  $M_{rem}$ , measured at 5 K and  $H_{ext} = 0$  after saturation in positive fields (up to  $10^4$  Oe) and subsequent application of a returning field  $H_{ret}$  [this procedure is shown schematically in panel (c)], adapted from Gillijns *et al.* [96].

compensation effect. As a consequence, the superconducting critical temperature  $T_c$  increased with increasing  $|H_{ext}|$  up to 5 kOe. Once the external field exceeds the saturation field  $H_s$  of the ferromagnet ( $H_s \simeq 5.5$  kOe at low temperatures), the domain structure in the ferromagnet disappears and the phase boundary abruptly returns back to the standard linear dependence  $(1 - T_c/T_{co}) \simeq |H_{ext}|/H_{c2}^{(0)}$ . Since the width and the shape of the magnetic domains continuously depend on the external field, the theory developed in section 2.2 is not directly applicable for the description of the experiment, although it qualitatively explains the main features of the OP nucleation in such S/F systems.

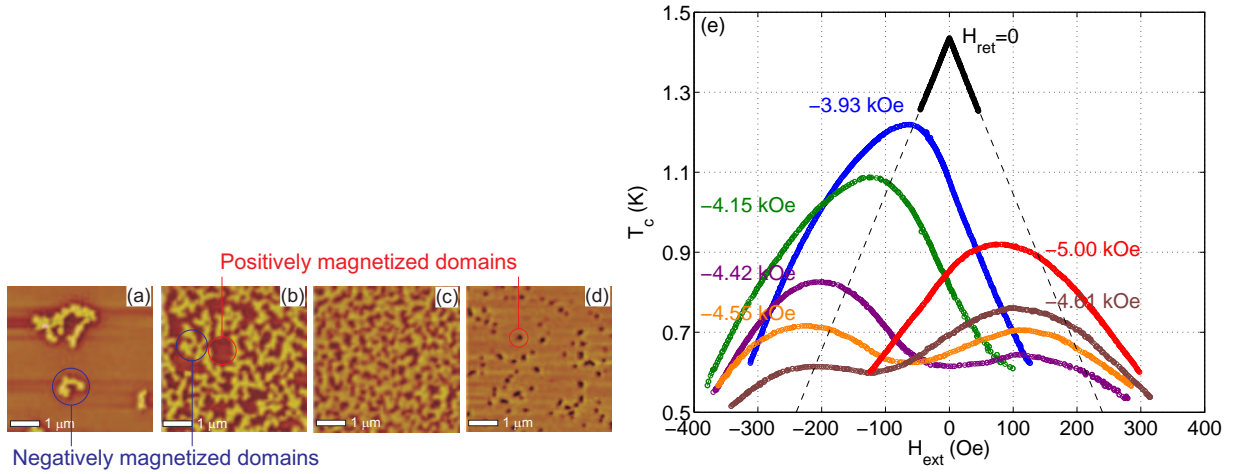
Substituting Nb by a superconductor with a smaller  $H_{c2}^{(0)}$  value (e.g., Pb with  $H_{c2}^{(0)} \simeq 1.8$  kOe) allows one to study the effect of the superconducting coherence length  $\xi_0 = \sqrt{\Phi_0/2\pi H_{c2}^{(0)}}$  on the localization of the OP. It was shown by Yang *et al.* [103] that the increase of the  $M_s/H_{c2}^{(0)}$  ratio suppresses the critical temperature of the formation of domain-wall superconductivity at zero external field<sup>‡</sup>, therefore superconductivity in Pb/BaFe<sub>12</sub>O<sub>19</sub> hybrids appeared only near the compensation fields above the reversed domains.

Direct visualization of localized superconductivity in Nb/PbFe<sub>12</sub>O<sub>19</sub> structures was performed by Fritzsche *et al.* [104]. The basic idea of this technique is the following: if the sample temperature becomes close to a local critical temperature at a certain position  $(x, y)$ , then a laser pulse, focused on that point, can induce the local destruction of superconductivity due to heating. The observed increase of the global resistance  $R$  of the superconducting bridge can be associated with the derivative  $dR(x, y)/dT$ . By varying the temperature and scanning the laser beam over the Nb bridge under investigation, it is possible to image the areas with different critical temperatures. For example, it allows one to attribute the formation of well-defined regions with rather high local critical temperatures above magnetic domains at the compensation field with the appearance of reversed-domain superconductivity.

The effect of the amplitude of the field modulation on the OP nucleation was considered by Gillijns *et al.* [105, 106] on thin-film trilayered hybrid F/S/F structures. In contrast to the BaFe<sub>12</sub>O<sub>19</sub> single crystal discussed above, the multilayered Co/Pd films are characterized by a high residual out-of-plane magnetization,  $M_s \sim 10^2$  Oe, almost independent on the external field at  $|H_{ext}| < H_{coer}$ , where  $H_{coer} \simeq 10^3$  Oe is the typical coercive field at low temperatures. The use of two ferromagnetic films with slightly different coercive fields allowed them to prepare different magnetic configurations and thus to control the amplitude of the nonuniform field

<sup>‡</sup> In section 2.3 we argued that the critical temperature,  $T_c^{(0)}$ , at zero external field is proportional to  $1 - 2\pi M_s/H_{c2}^{(0)}$ .





**Figure 7.** (color online) (a)–(d) MFM images obtained at  $T = 300$  K for  $H_{ret}$  values equal to  $-1.75$  kOe (a),  $-2.00$  kOe (b),  $-2.50$  kOe (c),  $-3.00$  kOe (d), the coercive field  $H_c^{300K} = 1.91$  kOe, adapted from Gillijns *et al.* [96]. The dark (bright) color represents domains with positive (negative) magnetization. (e) A set of experimental phase boundaries  $T_c(H_{ext})$  obtained for the same bilayered S/F sample (a superconducting Al film on top of a Co/Pt multilayer) in various magnetic states measured after the procedure of an incomplete demagnetization:  $H_{ext} = 0 \Rightarrow H_{ext} = 10$  kOe  $\Rightarrow H_{ext} = H_{ret} \Rightarrow H_{ext} = 0$  for various returning fields  $H_{ret}$  indicated on the diagram, the coercive field  $H_c^{5K} = 3.97$  kOe, adapted from Gillijns *et al.* [96].

inside the superconductor due to the superposition of the partial stray fields via an appropriate demagnetizing procedure. The effective doubling of the amplitude of the internal field for a configuration with two demagnetized ferromagnetic films (containing bubble domains) leads to a broadening of the temperature interval, where the  $T_c(H_{ext})$  line demonstrates the non-monotonous behavior. In other words, the critical temperature of domain-wall superconductivity at  $H_{ext} = 0$  expectedly decreases as the effective magnetization increases. In addition, the enhancement of the internal field results in a shift of the  $T_c$  maxima to higher  $H_{ext}$  values, what corresponds to reversed-domain superconductivity.

The spatial extension where the field compensation takes place, is a crucial parameter defining the nucleation of superconductivity: an OP trapped in a broader region results in a higher  $T_c$  value and vice versa (Gillijns *et al.* [96], Aladyshkin *et al.* [107]). The magnetic state of the ferromagnet can be reversibly changed after the following procedure of an incomplete demagnetization :  $H_{ext} = 0 \Rightarrow H_{ext} = H_s \Rightarrow H_{ext} = H_{ret} \Rightarrow H_{ext} = 0$ , where  $H_s$  is the saturation field (see Fig. 6). As a result, one can obtain any desirable remanent magnetization  $-M_s < M_{rem} < M_s$  (as well as any average width of the magnetic domains) by varying the  $H_{ret}$  value. At  $H_{ret} < 0$  the formation of the negative domains decreases the average width of the positive domains and it causes a drastic lowering of the height of the  $T_c$  peak, positioned at negative fields and attributed to the appearance of superconductivity above large positive domains [curves  $H_{ret} = -3.93$  kOe and  $H_{ret} = -4.15$  kOe in Fig. 7]. This observation is a direct consequence of the increase of the ground energy of the “particle-in-a-box” with decreasing width of the box. When  $M_{rem}$  is close to zero, thus indicating the presence of an equal distribution of positive and negative domains, a nearly symmetric phase boundary with two maxima of the same amplitude is recovered [curve  $H_{ret} = -4.55$  kOe in Fig. 7]. For higher  $H_{ret}$  values, the first peak, located at negative fields, disappears, whereas the peak at positive fields shifts up in temperature and is displaced to lower magnetic field values [curves  $H_{ret} = -4.61$  kOe and  $H_{ret} = -5.00$  kOe in Fig. 7]. This second peak eventually evolves into a linear phase boundary when the ferromagnetic film is fully magnetized in the negative direction.

#### OP nucleation in parallel magnetic field

We would like to mention a few related papers devoted to the nucleation of superconductivity in various planar S/F structures, where superconducting and ferromagnetic layers were not electrical insulated and thus an effect of exchange interaction can not be excluded.

An appearance of domain walls in permalloy film leading to dips in a field dependence of the resistivity

of a superconducting Nb film was observed by Rusanov *et al.* [108]. The position of these resistivity minima were found to be dependent on the sweep direction of the in-plane oriented external field. An opposite effect (maxima of resistivity at temperatures below  $T_{c0}$ ) was observed by Bell *et al.* [109] for thin-film amorphous S/F structures consisting of superconducting MoGe films and ferromagnetic GdNi layers. It was interpreted as flow of weakly pinned vortices induced by the stray field of magnetic domains in the ferromagnetic layers. Zhu *et al.* [110] demonstrated that the domain structure in multilayered CoPt films can be modified by applying in-plane external field during the deposition process. This deposition field does not change the overall perpendicular magnetic anisotropy of the Co/Pt films but it induces a weak in-plane magnetic anisotropy and eventually alters the domain patterns. Indeed, after demagnetizing with an in-plane ac magnetic field oriented along the deposition field direction, one can prepare the domain structure in a form of largely parallel stripe domains. In contrast to that, the same multilayered structure fabricated at zero field or demagnetized with out-of-plane ac field exhibits a nearly random labyrinth-type domain pattern. Sweeping the external field  $H_{ext}^{\parallel}$ , one can control the arrangement of domain walls, and drive the S/F hybrid from normal to superconducting state for the same temperature and magnetic field.

### 2.5. S/F hybrids with 2D periodic magnetic field: theory and experiments

In this section we continue the discussion concerning the properties of large-area S/F hybrids in which the nonuniform magnetic field is created by regular arrays of ferromagnetic dots. The fabrication of such magnetic structures makes it possible to achieve full control of the spatial characteristics of the nonuniform magnetic field (both the topology and the period), which can be eventually designed practically at will. One can expect that due to the field-compensation effect the inhomogeneous magnetic field modulated in both directions will affect the OP nucleation in the same way as for the planar S/F structures. However, the 2D periodicity of the magnetic field naturally leads to the appearance of well-defined commensurability effects for such hybrid systems. In other words, a resonant change in the thermodynamical and transport properties of the superconducting films appears as a function of the external magnetic field  $H_{ext}$ , similar to that observed for superconductors with periodic spatial modulation of their properties (e.g., perforated superconducting films [52, 53]). These matching phenomena take place at particular  $H_{ext,n}$  values that can be used as indicators allowing us to find a relationship between the most probable microscopic arrangement of the vortices in the periodic potential and the global characteristics of the considered S/F hybrids measured in the experiments.

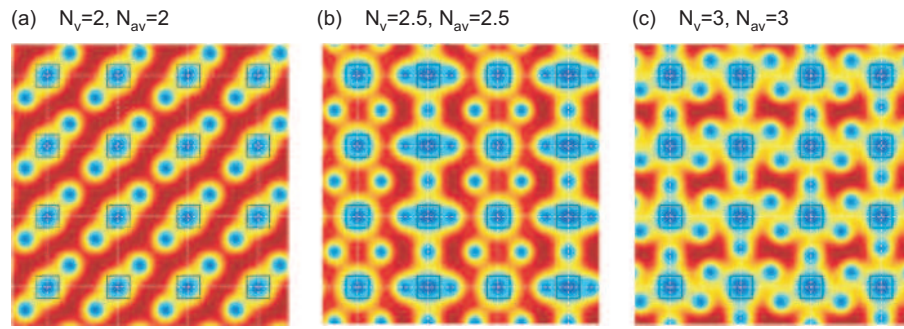
#### *Commensurate solutions of the GL equations.*

The periodic solutions of the GL equations in the presence of a nonuniform 2D periodic magnetic field can be constructed by considering one or more magnetic unit cells (of total area  $S$ ) and applying the following boundary conditions:

$$\begin{aligned} \mathbf{A}(\mathbf{r} + \mathbf{b}_k) &= \mathbf{A}(\mathbf{r}) + \nabla\eta_k(\mathbf{r}), \\ \Psi(\mathbf{r} + \mathbf{b}_k) &= \Psi(\mathbf{r}) \exp(2\pi i\eta_k(\mathbf{r})/\Phi_0), \end{aligned} \quad (15)$$

where  $\mathbf{b}_k$ ,  $k = \{x, y\}$  are the lattice vectors, and  $\eta_k$  is the gauge potential (Doria *et al.* [111]). The gauge transformation Eq. (15) is possible provided that the flux induced by the external magnetic field  $H_{ext}S$  through the chosen area  $S$  is equal to an integer number of flux quanta,  $n\Phi_0$ , which gives us the matching fields  $H_n^{ext} = \pm n\Phi_0/S$ .

The formation of different vortex patterns in S/F hybrids, containing square arrays of the magnetic dots with perpendicular magnetization, at  $H_{ext} = 0$  were studied by Priour and Fertig [112, 113] and Milošević and Peeters [114, 115]. Since the total flux through an underlying superconducting film, infinite in the lateral direction, equals to zero, vortices cannot nucleate without corresponding antivortices, keeping the total vorticity zero. It was shown that the number of the vortex-antivortex pairs depends on the dipolar moment of the magnetic dot. The equilibrium vortex phase, corresponding to the minimum of the free energy functional, can exhibit a lower symmetry than the symmetry of the nonuniform magnetic field. In order to get such vortex states of reduced symmetry, the GL equations with periodic boundary conditions should be considered in a large supercell ( $2 \times 2$ ,  $4 \times 4$  etc.). In most cases vortices are confined to the dot regions, while the antivortices, depending on the dot's magnetization, can form a rich variety of regular lattice states with broken orientational and mirror symmetries (Fig. 8). The creation of vortex-antivortex pairs in case of ferromagnetic dots with



**Figure 8.** (color online) Contour-plots of the Cooper pair density  $|\psi|^2$  of stable vortex phases in superconducting thin films in the presence of a magnetic dot array with dipolar moments oriented perpendicular to the film plane, adapted from Priour and Fertig [112]. The images in (a), (b), and (c) correspond to magnetic dot strengths of 3.61, 4.33, and 4.95 fundamental flux units, respectively (the dot's dipole moment is specified by the positive flux passing through each unit cell). The Cooper pair density is highest in red and lowest in blue regions. Antivortices appear as small blue spots between the magnetic dots depicted by black squares. The number of vortices  $N_v$  and antivortices  $N_{av}$  per unit cell are indicated on the plots. The calculations were performed for a large  $4 \times 4$  supercell with periodic boundary conditions and for the following parameters:  $\kappa = \lambda/\xi = 4$ , the period of the magnetic dot array is  $6.25 \xi$ , the lateral size of the magnetic dot is  $2\xi \times 2\xi$ .

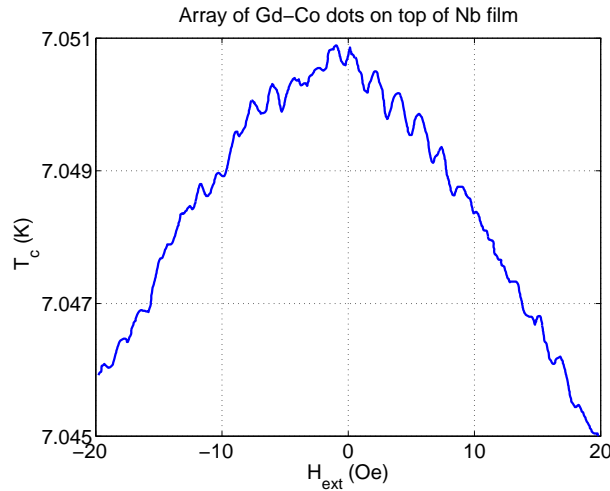
in-plane magnetization was considered by Milošević and Peeters [116]. As expected, vortex-antivortex pairs appear under the poles of each magnet according to their specific inhomogeneous magnetic field keeping the total flux through the superconducting film equals to zero.

The influence of the external field on the formation of symmetrical and asymmetrical commensurate vortex configurations in thin superconducting films in the presence of a square array of out-of-plane magnetized dots was investigated by Milošević and Peeters [114, 115, 117, 118]. The simulations were carried out only for some discrete values  $H_{ext,n}$  of the external field, corresponding to the magnetic flux quantization per magnetic supercell of area  $S$ . Vortices were shown to be attracted by the magnetic dots in the parallel case (at  $H_{ext,n} > 0$  for  $M_z > 0$ ) and repelled in the antiparallel case (at  $H_{ext,n} < 0$  for  $M_z > 0$ ). In the parallel case the vortex configurations for the integer matching fields are similar to that for the vortex pinning by regular arrays of antidots.

The temperature dependence of the magnetization threshold for the creation of vortex-antivortex pairs was considered in Ref. [118]. It was noted that the system will not necessarily relax to the ground state, if there are metastable states, corresponding to local minima in the free energy. As long as the given vortex state is still stable and it is separated from other stable vortex configurations by a finite energy barrier (analogous to the Bean-Livingston barrier for the vortex entry into superconducting samples), then the vorticity remains the same even when changing temperature. However, an increase in temperature resulting in a decrease of the height of the energy barriers and strengthening of the thermal fluctuations, can eventually cause a phase transition between the vortex states with different number of vortices. The modification of the ground state (at  $H_{ext} = 0$ ) by the creation of extra vortex-antivortex pairs, when changing temperature and/or increasing the  $M_s$  value manifests itself as cusps in the phase boundary separating superconductor from the normal metal phase in the  $M_s - T$  diagram, similar to the Little-Parks oscillations in the  $T_c(H_{ext})$  dependence [55, 56].

#### *Oscillatory nature of the phase transition line (in-plane dot's magnetization)*

The influence of two-dimensional square arrays of micron-sized, in-plane magnetized particles (SmCo, GdCo, FeNi) on the electrical resistance of a superconducting Nb film, usually interpreted as field-induced variations of the critical temperature, were experimentally studied by Pannetier *et al.* [97], Otani *et al.* [119], Geoffroy *et al.* [120]. The oscillatory dependence of the resistivity  $\rho$  on the perpendicularly oriented external field  $H_{ext}$  with a period  $\Delta H_{ext}$  close to  $\Phi_0/a^2$  was observed at  $T < T_{c0}$  only when the dots had been magnetized before cooling (here  $a$  is the period of the magnetic dot array). The appearance of minima in the  $\rho(H_{ext})$  dependence, which are reminiscent of the Little-Parks oscillations for multiply-connected superconductors and superconducting networks (see the monograph of Berger and Rubinstein [49] and references therein), were attributed to the



**Figure 9.** (color online) Phase transition line of a superconducting Nb film with a ferromagnetic  $\text{Gd}_{33}\text{Co}_{67}$  particle array (the period  $4\mu\text{m}$ ), adapted from Otani *et al.* [119].

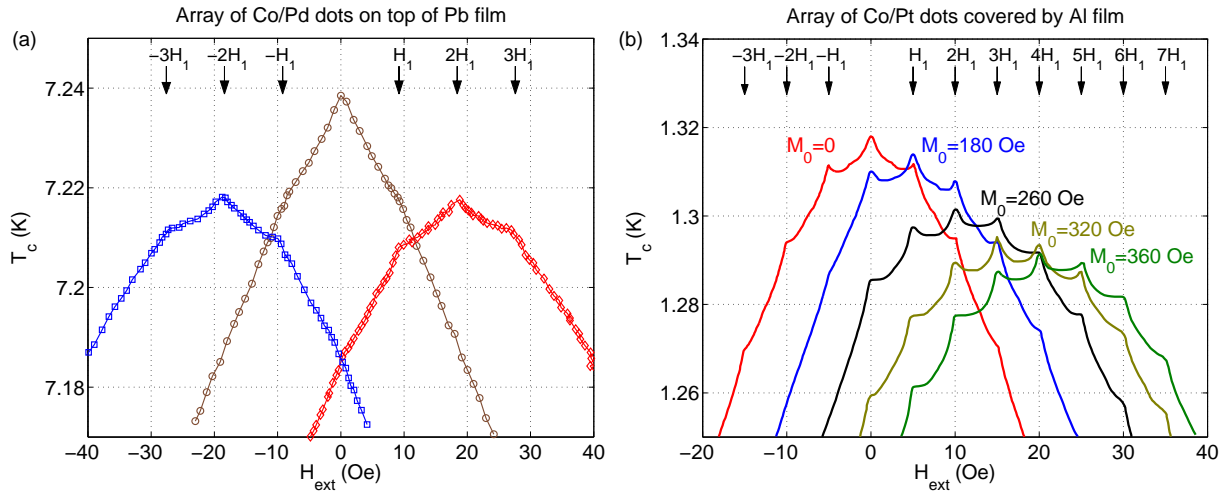
variation of the critical temperature,  $T_c = T_c(H_{ext})$  due to the fluxoid quantization in each magnetic unit cell.

#### *Tunable field-induced superconductivity for arrays of out-of-plane magnetized dots*

The fabrication of periodic arrays of multilayered Co/Pd and Co/Pt magnetic dots with out-of-plane magnetic moments allows one to observe both the matching phenomena and the modified OP nucleation due to the field-compensation effect. The stray field induced by positively magnetized dots has a positive  $z$ -component of the magnetic field under the dots and a negative one in the area in between the dots. Thus, the magnetic field in the region in between the dots will be effectively compensated at  $H_{ext} > 0$ , stimulating the appearance of superconductivity at nonzero  $H_{ext}$  values (magnetic field-induced superconductivity). Lange *et al.* [121] demonstrated that the  $T_c$  maximum is located at  $H_{ext} = 0$  for the demagnetized magnetic dots and it is shifted towards a certain  $H_{ext,n}$  which depends on the dot's magnetization [see the panel (a) in Fig. 10]. This quantized shift of the  $T_c$  was attributed to the field compensation in the interdot areas accompanied by an annihilation of the interstitial antivortices under the action of the external field, since (i) the number of antivortices is determined by the magnetic moment of the dots; (ii) the interstitial antivortices can be fully cancelled only at the matching fields  $H_{ext,n} = n\Phi_0/a^2$  (Milošević and Peeters [122]). Thus, the appearance of periodic kinks in the  $T_c(H_{ext})$  phase boundary with a period coinciding with the first matching field  $H_1$  can be associated with the fluxoid quantization, confirming that superconductivity indeed nucleates in multiply connected regions of the film.

The results of further investigations on similar hybrid systems (an array of micron-sized Co/Pt dots on top of an Al film) was presented by Gillijns *et al.* [123, 124, 125]. As a consequence of the rather large diameter of the dots, the demagnetized dot's state microscopically corresponds to a magnetic multidomain state with very weak stray field. As it was demonstrated in Ref. [123], the remanent magnetization of the dots, which were initially demagnetized, depends monotonously on the maximal applied field (excursion field)  $H_{ret}$ . Thus the total remanent magnetic moment of the dot becomes variable and tunable, hereby changing the influence of the ferromagnet on the superconductor in a continuous way. It was found that a gradual increase of the dot's magnetization from zero to a certain saturated value results (i) in a quantized displacement of the main  $T_c$  maximum towards  $nH_1$  ( $n$  is integer) due to the quantized character of the field-induced superconductivity [the panel (b) in Fig. 10]; (ii) in an enhancement of the local  $T_c$  maxima, attributed to the formation of a commensurate vortex phase at discrete matching fields, which becomes more pronounced as compared to Ref. [121].

The effect of changing the average remanent magnetization  $M_{rem}$  and the radius  $R_f$  of the magnetic dots on the superconducting properties of an Al film deposited on top of a periodic array of such dots was studied by Gillijns *et al.* [124, 125]. Indeed, once the dot's magnetization becomes saturated, the only way to further



**Figure 10.** (color online) Field-induced superconductivity in a superconducting film with an array of magnetic dots:

(a) The  $T_c(H_{ext})$  dependences obtained for a Pb film after demagnetization of the ferromagnetic dot array (the brown central curve marked by circles), saturation of the dots in a large positive  $H_{ext}$  (the red right curve marked by diamonds), and saturation in a large negative  $H_{ext}$  (the blue left curve marked by squares), adapted from Lange *et al.* [121]. The period of the lattice was  $1.5 \mu\text{m}$ . The arrows depict the corresponding matching fields.

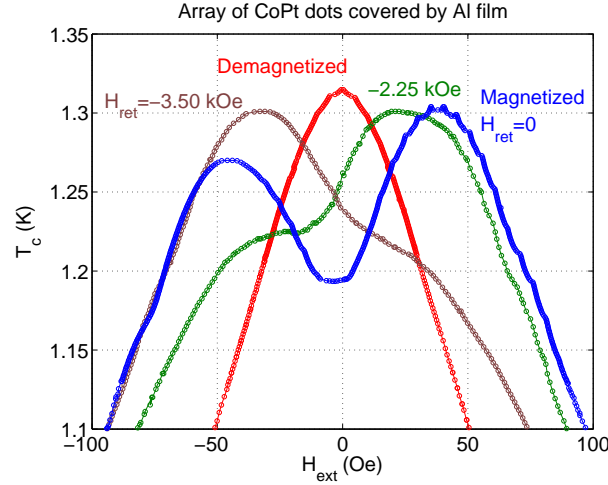
(b) Superconducting transition  $T_c(H)$  of an Al film for different magnetic states of the square array of the ferromagnetic dots of the period  $2 \mu\text{m}$ , adapted from Gillijns *et al.* [123]. By increasing the magnetization a clear shift of  $T_c(H_{ext})$  and a decrease of  $T_c^{max}$  is observed.

increase the magnetic flux from each magnet can be achieved by increasing the lateral size of the dots. It was experimentally found that the larger the  $R_f$  value, the smaller the necessary  $\Delta M$  needed to shift the main  $T_c(H_{ext})$  maximum by one matching field.

Both the field compensation and matching effects in plain Al films with a square array of ferromagnetic Co/Pt disks were investigated by Gillijns *et al.* [96] and Aladyshkin *et al.* [107]. Due to the presence of the out-of-plane magnetized dots, there are *three* different areas, where the OP can potentially nucleate: above the positive or negative domains, inside the magnetic dot, and in between the dots. In the demagnetized state the interdot field is close to zero, therefore superconductivity starts to nucleate at this position at relatively low magnetic fields, resulting in an almost linear phase boundary centered at  $H_{ext} = 0$ . By magnetizing the dots positively (i) the amplitude of the field between the dots increases negatively and (ii) the typical width of the positive domains becomes larger than that for negative domains. Therefore the peak, associated with the OP localization between the dots, shifts towards positive fields. In addition, a second local  $T_c$  maximum, corresponding to the appearance of superconductivity above the broader positive domains, appears, while the OP nucleation above narrower negative domains is still suppressed. For negatively magnetized dots the reversed effect occurs. It is important to note, that the amplitude of the main  $T_c$  peak, corresponding to the OP nucleation between the dots, remains almost constant, since the mentioned area of the OP localization is almost independent of the dot's magnetic state (Fig. 11).

#### Individual ferromagnetic dots above/inside superconducting films

Marmorkos *et al.* [126] studied a possibility to create giant vortices by a ferromagnetic disk with out-of-plane magnetization embedded in thin superconducting film within full nonlinear, self-consistent Ginzburg-Landau equations. Later using the same model Milošević and Peeters [127, 128, 129] considered the formation of vortex-antivortex structures in plain superconducting films, infinite in the lateral direction, in the field of an isolated ferromagnetic disk with out-of-plane magnetization within the full nonlinear Ginzburg-Landau theory. Antivortices were shown to be stabilized in shells around a central core of vortices (or a giant vortex) with magnetization-controlled "magic numbers" (Fig. 12). The transition between the different vortex phases while varying the parameters of the ferromagnetic dot (namely, the radius and the magnetization) occurs through the

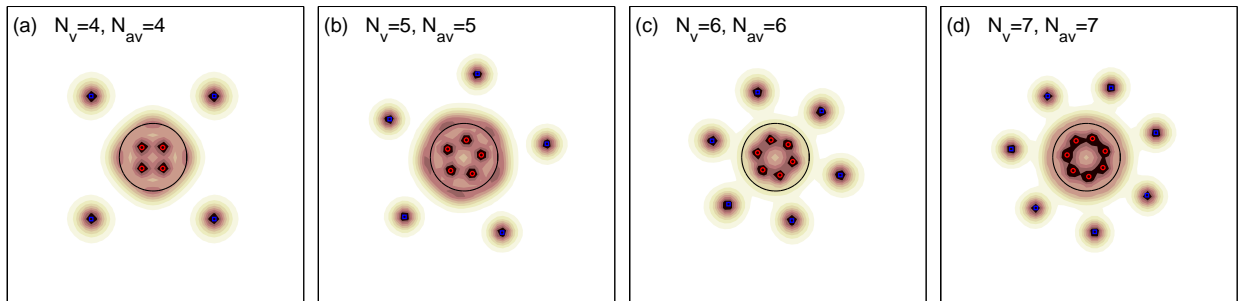


**Figure 11.** (color online) The phase boundaries  $T_c(H_{ext})$  for an S/F hybrid, consisting of an Al film and an array of magnetic dots, in the demagnetized state, in the completely magnetized state in positive direction as well as in several intermediate magnetic states, adapted from Gillijns *et al.* [96] and Aladyshkin *et al.* [107]. The period  $\Delta H_{ext}$  of the  $T_c$  oscillations, which are distinctly seen in the curve corresponding to the magnetized states, is equal to 5.1 Oe and it exactly coincides with the matching field, i.e.,  $\Delta H_{ext} = \Phi_0/S$ , where  $\Phi_0 = 2 \cdot 10^{-7}$  Oe $\cdot$ cm $^2$  is the flux quantum and  $S = 4 \mu\text{m}^2$  is the area of the unit cell. Note that field and temperature intervals shown here are much broader than those presented in Fig. 10 (b).

creation of a vortex-antivortex pair under the magnetic disk edge.

## 2.6. Mesoscopic S/F hybrids: theory and experiments

In all the description of nucleation of superconductivity so far, we have ignored the effects of the sample's borders. It is well known that the OP patterns in mesoscopic superconducting samples with lateral dimensions comparable to the superconducting coherence length and magnetic penetration depth, is substantially influenced by the geometry of the superconductor (see review of Chibotaru *et al.* [40] and references therein). As a result, the presence of the sample's boundaries allows the appearance of exotic states (giant vortex states, vortex clusters, shell configurations etc.), otherwise forbidden for bulk superconductors and non-patterned plain



**Figure 12.** (color online) Contour-plots of the Cooper pair density  $|\psi|^2$ , illustrating the appearance of vortex-antivortex shell structures in large-area superconducting film in the field of perpendicularly magnetized disk for different magnetic moments  $m$  of the ferromagnetic particle:  $m/m_0 = 25$  (a), 29 (b), 35 (c) and 38 (d), where  $m_0 = H_{c2} \xi^3$ , by courtesy of M.V. Milošević. The highest  $|\psi|$  values are shown in lighter shades and the lowest densities in darker shades. It is important to note that only the central part of the superconducting film ( $45 \xi \times 45 \xi$ ) is shown here, black solid line schematically depicts the edge of the magnetic dot. Red circles correspond to the vortex cores, while blue squares mark the position of antivortices. The number of vortices  $N_v$  and antivortices  $N_{av}$  are indicated on the plots. The simulations were performed for the following parameters:  $\kappa = \lambda/\xi = 1.2$ , the lateral size of superconducting sample is  $256 \xi \times 256 \xi$ , the radius of ferromagnetic disk is  $4.53 \xi$ , and the thicknesses of superconductor and ferromagnet are equal to  $0.1 \xi$ .



superconducting films. Since, as we have pointed out above, a nonuniform magnetic field is an alternative way to confine the superconducting OP in a certain  $H_{ext}$  and  $T$  range, mesoscopic S/F hybrids seem to be of interest for studying the interplay between different mechanisms of confinement of the superconducting condensate.

It is important to note, that the screening effects can still be omitted provided that the lateral size of the thin superconducting sample is smaller than the effective penetration depth  $\lambda_{2D} = \lambda^2/D_s$ . In this case the self-interaction of the superconducting condensate can be taken into account solving the nonlinear decoupled GL equation

$$-\xi^2 \left[ \nabla + \frac{2\pi i}{\Phi_0} \mathbf{A}(\mathbf{r}) \right]^2 \psi - \psi + |\psi|^2 \psi = 0, \quad (16)$$

where the vector potential distribution is given by the external sources and the ferromagnet only [see Eq. (6)].

#### *Interplay between different regimes of the OP nucleation*

As we anticipated above, a very interesting phenomenon in mesoscopic S/F hybrids is the interplay between competing regimes of the OP nucleation, which can be clearly seen in the case of a small-sized magnetic dot of radius  $R_f$  placed above a mesoscopic superconducting disk,  $R_f \ll R_s$ . Indeed, the  $|\psi|$  maximum can be generally located either at the central part of the superconducting disk, close to the magnetic dot (magnetic-dot-assisted superconductivity) or at the outer perimeter of the superconducting disk (surface superconductivity). For a positively magnetized dot the regime of the magnetic-dot-assisted superconductivity, associated with the appearance of superconductivity in the region with compensated magnetic field, can be realized only for  $H_{ext} < 0$ . At the compensation field ( $|H_{ext}| \simeq B_0$ ) and provided that  $\sqrt{\Phi_0/(2\pi B_0)} \ll R_s$  the enhancement of the  $z$ -component of the field near the disk edge acts as a magnetic barrier for the superconducting condensate and it prevents the edge nucleation of superconductivity even in small-sized superconductors ( $B_0$  being the maximum of the self-field of the magnetized dot). The OP nucleation near the magnetic dot becomes possible, if the critical temperature  $T_c^{(0)}$  of the formation of localized superconductivity with the OP maximum at the superconducting disk center

$$1 - \frac{T_c^{(0)}}{T_{c0}} \simeq \frac{2\pi\xi_0^2}{\Phi_0} |H_{ext} + B_0|.$$

exceeds the critical temperature for the edge nucleation regime

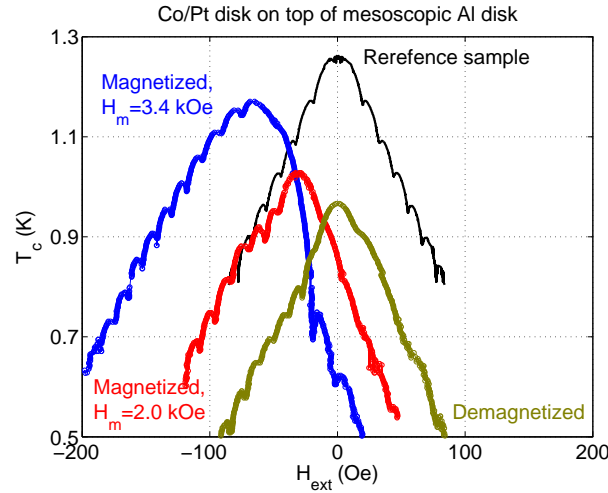
$$1 - \frac{T_{c3}}{T_{c0}} \simeq 0.59 \frac{2\pi\xi_0^2}{\Phi_0} |H_{ext}|,$$

corresponding to the critical field of surface superconductivity  $H_{c3} = 1.69 H_{c2}$  [43, 44, 45]. Due to the different slopes  $dT_c^{(0)}/dH_{ext}$  and  $dT_{c3}/dH_{ext}$  and the offsets, one can conclude that the edge OP nucleation regime apparently dominates both for positive and large negative  $H_{ext}$  values. Only in the intermediate field range the highest critical temperature corresponds to the formation of superconductivity near the magnetic particle.

#### *Little-Parks oscillations in mesoscopic samples*

The nucleation of superconductivity in axially-symmetrical mesoscopic S/F structures (e.g., superconducting disks or rings in the field of a perpendicularly magnetized ferromagnetic circular dot) were studied theoretically by Aladyshkin *et al.* [54], Cheng and Fertig [130], Milošević *et al.* [131, 132], and experimentally by Golubović *et al.* [133, 134, 135, 136] and Schildermans *et al.* [137]. Due to the cylindrical symmetry of the problem, superconductivity was found to appear only in the form of giant vortices  $\psi(r, \theta) = f_L(r) \exp(iL\theta)$ , where  $L$  is the angular momentum  $L$  of the Cooper-pairs (vorticity). The appearance of vortex-antivortex configurations in superconducting disks of finite radius at temperatures close to  $T_c$  is possible, although these states were predicted to be metastable states.

The observed periodic cusp-like behavior of the  $T_c(H_{ext})$  dependence was attributed to the field-induced transition between states with different vorticity similar to that of mesoscopic superconductors in a uniform magnetic field. However, the stray field, induced by the magnetized dot, was shown to be responsible for a peculiar asymmetry of the oscillatory  $T_c(H_{ext})$  phase boundary and a shift of the main  $T_c$  maximum towards nonzero  $H_{ext}$  values [54, 133, 134, 135]. The mentioned abrupt modification of the preferable nucleation regime when sweeping  $H_{ext}$  can lead to a double change in the slope of the  $T_c(H_{ext})$  envelope from  $T_{c0}/H_{c3}^{(0)}$  to  $T_{c0}/H_{c2}^{(0)}$  [54, 137] (see Fig. 13). The restoration of the slope close to  $T_{c0}/H_{c2}^{(0)}$  can be interpreted as an effective



**Figure 13.** (color online) The phase transition lines  $T_c(H_{ext})$ , obtained experimentally for the mesoscopic S/F hybrid system with a  $0.1 R_n$  criterion for three different magnetic states (completely magnetized, partly magnetized and demagnetized states),  $R_n$  and  $H_m$  being the normal-state resistance and the magnetizing field, adapted from Schildermans *et al.* [137]. The considered S/F system consists of superconducting Al disk covered by ferromagnetic Co/Pt multilayered film of the same radius  $R_s = R_f = 0.825 \mu\text{m}$ . Black solid line represents the  $T_c(H_{ext})$  dependence for the reference Al disk of the same lateral size.

elimination of the boundary effects in mesoscopic S/F samples at the compensation field (near the main  $T_c$  maximum). Interestingly, the nonuniform magnetic field can be used to control the shift in the field dependence of the maximal critical current  $I_c(H_{ext})$  for a bias current flowing through the superconducting loop, which allows one to tune the internal phase shift in superconducting networks (Golubovic *et al.* [136]).

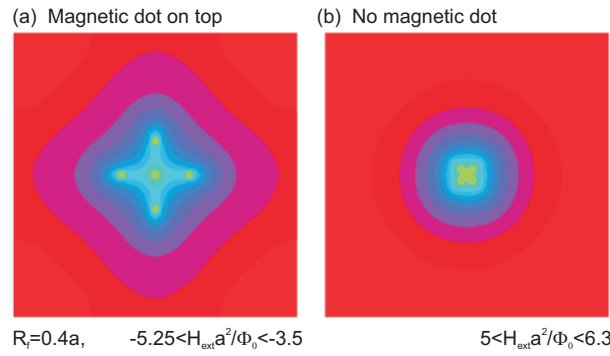
It is important to note that the periodicity  $\Delta H_{ext}$  of the Little-Parks oscillations in the  $T_c(H_{ext})$  dependence is explicitly given by the area where the superconducting OP is confined. For edge nucleation only the area enclosed by the superconductor determines the period of the oscillations, which can be roughly estimated as  $\Delta H_{ext} \simeq \Phi_0/R_s^2$ . However in the case of the magnetic-dot-assisted nucleation (in the vicinity of the compensation field) the area of the OP localization is determined by the spatial characteristics of the nonuniform magnetic field (either the dot's radius  $R_f$  or the vertical separation between the dot and the superconductor  $Z_d$ ), therefore  $\Delta H_{ext} \simeq \Phi_0/\max\{R_f^2, Z_d^2\}$ . As a consequence, the change of the nucleation regimes manifests itself as an abrupt modification of the oscillatory  $T_c(H_{ext})$  dependence. In particular, both the amplitude and the period of the Little-Parks oscillations become much larger, provided that  $(Z_d, R_f) \ll R_s$  and the OP wave function localizes far from the sample's edges (Aladyshkin *et al.* [54], Carballeira *et al.* [138]).

#### *Symmetry-induced vortex-antivortex patterns*

Mesoscopic S/F hybrids of a reduced symmetry (e.g., structures consisting of a superconductor/ferromagnet disks and/or regular polygons) represent nice model systems for studying symmetry-induced phenomena. The formation of different vortex-antivortex configurations was studied theoretically by Carballeira *et al.* [138] and Chen *et al.* [139] for mesoscopic superconducting squares with a circular ferromagnetic dot magnetized perpendicularly, and experimentally by Golubovic *et al.* [140] for a superconducting Al disk with a magnetic triangle of Co/Pt on top. It was shown that the symmetry-consistent solutions of the Ginzburg-Landau equations<sup>†</sup>, earlier predicted for mesoscopic superconducting polygons by Chibotaru *et al.* [40], are preserved for regular superconducting polygons in the stray field of ferromagnetic disk. However, since spontaneously formed vortices and antivortices interact with the magnetic dot in a different way, it leads to a modification of the symmetry-induced vortex patterns (see Fig. 14). In particular, the dot can be used to enlarge these vortex-antivortex patterns, thus facilitating their experimental observation with local vortex-imaging techniques

<sup>†</sup> By symmetry-consistent solutions we mean those vortex configurations reflecting the symmetry of the problem. For instance, in a mesoscopic superconducting square with vorticity  $L = 3$ , the state consisting of four vortices and a central antivortex may have lower energy than the configuration of three equidistant vortices, which breaks the square symmetry (Chibotaru *et al.* [141]).





**Figure 14.** (color online) A comparison between the vortex-antivortex patterns that can be observed for  $L = 3$  (a) with and (b) without the magnetic dot on top of the superconducting square,  $a$  is the lateral size of the sample, adapted from Carballeira *et al.* [138]. As can be seen, the vortex-antivortex pattern rotates  $45^\circ$  and expands dramatically in the presence of the magnetic dot.

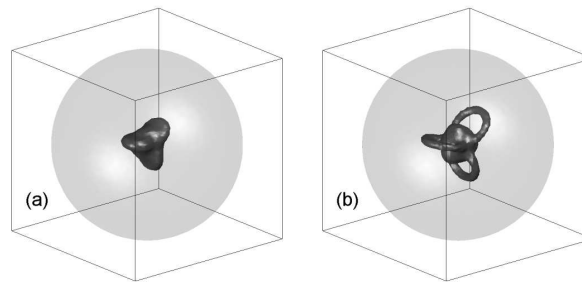
(“magnetic lensing”).

#### *Embedded magnetic particles*

Doria [142, 143] and Doria *et al.* [144, 145] studied theoretically the formation of vortex patterns induced by magnetic inclusions embedded in a superconducting material but electrically insulated from the multiply-connected superconductor. Since in the absence of an external field, flux lines should be closed, vortex lines are expected to start and end at the magnetic inclusions. The calculations, performed in the framework of GL theory for a mesoscopic superconducting sphere with a single magnetic point like particle in its center, reveal that the confined vortex loops arise in triplets from the normal core when the magnetic moment reaches the scale defined by  $m_0 = \Phi_0 \xi / 2\pi$ . Therefore a vortex pattern is made of confined vortices, loops and also broken loops that spring to the surface in the form of pairs. This vortex state provides a spontaneous vortex phase scenario for bulk superconductors with magnetic inclusions, where the growth of the vortex loops interconnects neighboring magnetic inclusions.

#### *Effect of the finite thickness of the superconducting film*

The effect of the finite thickness of the superconducting film on the  $T_c(H_{ext})$  dependence was studied theoretically by Aladyshkin *et al.* [54], Aladyshkin and Moshchalkov [95], and Schildermans *et al.* [137]. It follows from Maxwell’s equations that the faster the spatial variation of the magnetic field in the lateral  $(x, y)$ –directions, the faster the decay of the field in the transverse  $z$ –direction. However the OP variation along the sample’s thickness can be effectively suppressed by a requirement of vanishing the normal derivative  $\partial\psi/\partial n$  of the superconducting OP at the top and bottom surfaces of the superconductor. Indeed, the



**Figure 15.** (color online) Confined vortex loops arise in sets of threes inside a mesoscopic superconducting sphere (radius  $15 \xi$ ), as shown here for two consecutive values of the point-like magnetic moment that occupies its center: (a)  $m/m_0 = 15$ ; (b)  $m/m_0 = 20$ , by the courtesy of M. M. Doria.

external field applied along the  $z$ -direction makes the OP variations over the superconducting film thickness more energetically unfavorable than in the lateral direction, especially for  $H_{ext}$  values of the order of the compensation field or higher. The field dependence  $T_c(H_{ext})$  for rather thick superconducting films in the high-field limit is similar to the phase transition line described by Eq. (7). This behavior results from the effective averaging of the inhomogeneous magnetic field by the quasi-uniform OP wave function over the sample thickness, which substantially weakens the effect arising from the field modulation in the lateral direction. In other words, only the lateral inhomogeneity leads to the anomalous  $T_c(H_{ext})$  dependence and reentrant superconductivity in particular, while the transverse field non-uniformity masks this effect. Interestingly, the location where superconductivity starts to nucleate shifts towards the point with zero total magnetic field (not only  $z$ -component of the magnetic field) as the thickness of superconductor increases (Aladyshkin *et al.* [54]).

### 3. Vortex matter in non-uniform magnetic fields at low temperatures

#### 3.1. London description of a magnetically coupled S/F hybrid system

In section 2 we have reviewed the theoretical modelling within the GL formalism and the experimental data concerning the superconducting properties of S/F hybrids rather close to the phase transition line  $T_c(H_{ext})$ . In this section we consider the properties of S/F hybrids for a fully developed superconducting OP wave function, i.e. for  $T \ll T_{c0}$ . In this limit, the superconducting properties of the S/F hybrids can be correctly described by the London theory, omitting any spatial variations of the OP. As before, we assume that the coercive field of the ferromagnet is much higher than the upper critical field of the superconductor. This guarantees that, during the investigation of the superconducting properties, no changes in the ferromagnetic element(s) will occur.

At rather low temperatures one should take into account that the magnetic field, induced by screening (Meissner) currents or by vortices, can no longer be neglected and will strongly interact with the ferromagnet. The free energy functional of the S/F hybrid can be written in the following form [44, 45]

$$G_{sf} = G_{s0} + G_m + \int_V \left\{ \frac{\lambda^2}{8\pi} (\text{rot } \mathbf{B})^2 + \frac{\mathbf{B}^2}{8\pi} - \mathbf{B} \cdot \mathbf{M} - \frac{\mathbf{B} \cdot \mathbf{H}_{ext}}{4\pi} \right\} dV. \quad (17)$$

As before,  $G_{s0}$  is the self-energy of superconductor, while the term  $G_m$  depends on the magnetization of ferromagnet only. Assuming that  $G_m$  is constant for a given distribution of magnetization and minimizing this functional  $G_{sf}$  with respect to the vector potential  $\mathbf{A}$ , one can obtain the London-Maxwell equation:

$$\mathbf{B} + \lambda^2 \text{rot rot } \mathbf{B} = \Phi_0 \sum_i \delta(\mathbf{r} - \mathbf{R}_{v,i}) \mathbf{z}_0 + 4\pi\lambda^2 \text{rot rot } \mathbf{M}, \quad (18)$$

where summation should be done over all vortices, positioned at points  $\{\mathbf{R}_{v,i}\}$  and  $\delta(r)$  is the Dirac delta-function<sup>†</sup>. In Eq. (18) we assumed that each vortex line, parallel to the  $z$ -axis and carrying one flux quantum, generates a phase distribution  $\Theta = \varphi$  with  $\text{rot}(\nabla\Theta) = \text{rot}([\mathbf{z}_0 \times \mathbf{r}_0]/r) = \delta(r)\mathbf{z}_0$  [here  $(r, \varphi, z)$  are cylindrical coordinates with the origin chosen at the vortex]. It should be noticed that the solution of Eq. (18) gives the magnetic field distribution for a given vortex configuration, and in order to find the field pattern one should find the minimum of the total free energy  $G_{sf}$  with respect to the vortex positions  $\{\mathbf{R}_{v,i}\}$ .

#### 3.2. Interaction of a point magnetic dipole with a superconductor

Next we consider the generic problem of the interaction of a superconductor with a point magnetic dipole positioned at a height  $Z_d$  above the superconducting sample. Introducing the OP phase, which determines the density of supercurrents  $\mathbf{j}_s = c/(4\pi\lambda^2) [\Phi_0/(2\pi)\nabla\Theta - \mathbf{A}]$ , the free energy functional Eq. (17) at  $H_{ext} = 0$  can be rewritten as  $G_{sf} = G_{s0} + G_m + G_s$ , where

$$G_s = \int \left\{ \frac{\Phi_0^2}{32\pi^3\lambda^2} (\nabla\Theta \cdot \nabla\Theta) - \frac{\Phi_0}{16\pi^2\lambda^2} (\nabla\Theta \cdot \mathbf{A}) - \frac{1}{2} \mathbf{B} \cdot \mathbf{M} \right\} dV \quad (19)$$

similar to that obtained by Erdin *et al.* [146]. Due to the linearity of Eq. (18), we can introduce the field  $\mathbf{B}_v = \text{rot } \mathbf{A}_v$ , generated by a single vortex line positioned at  $\mathbf{r} = \mathbf{R}_v$ , and the field  $\mathbf{B}_m = \text{rot } \mathbf{A}_m$ , corresponding to the screening (Meissner) current distribution induced by the magnetic dipole<sup>‡</sup>. As a result, for the particular case of a point magnetic dipole,  $\mathbf{M} = \mathbf{m}_0 \delta(\mathbf{r} - \mathbf{R}_d)$ , the energy of the system  $G_s$  depending on the supercurrents can be represented as a sum of the self-energy of the vortex  $\epsilon_v^{(0)} = (\Phi_0/4\pi\lambda)^2 \ln \lambda/\xi$ , which is independent on the dipolar moment, and a term  $G_{int}$ , responsible for the interaction between the dipole and superconductor (e.g., Wei *et al.* [147], Carneiro [148]):

$$G_{int} = -\frac{1}{2} \mathbf{m}_0 \cdot \mathbf{B}_m(\mathbf{R}_d) - \mathbf{m}_0 \cdot \mathbf{B}_v(\mathbf{R}_d). \quad (20)$$

<sup>†</sup> We follow the standard definition of the  $\delta$ -function:  $\delta(x, y) = 0$  for any  $x \neq 0$ ,  $y \neq 0$ , and  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x, y) dx dy = 1$ .

<sup>‡</sup> These fields  $\mathbf{B}_v$  and  $\mathbf{B}_m$  are the solutions of the following differential equations

$$\begin{aligned} \mathbf{B}_v + \lambda^2 \text{rot rot } \mathbf{B}_v &= \Phi_0 \delta(\mathbf{r} - \mathbf{R}_v) \mathbf{z}_0, \\ \mathbf{B}_m + \lambda^2 \text{rot rot } \mathbf{B}_m &= 4\pi\lambda^2 \text{rot rot } \mathbf{M}. \end{aligned}$$

The first term in Eq. (20) corresponds to the interaction between the magnetic dipole and the local field at the dipole's position  $\mathbf{B}_m(\mathbf{R}_d)$  due to the screening currents. The second term in Eq. (20) gives the energy of the interaction between the vortex and the magnetic dipole, which consists of two parts: (i) the “hydrodynamical” interaction between circulating supercurrents due to the vortex on one hand and the dipole on the other hand; (ii) the interaction between the stray field of the vortex with the magnetic moment. Interestingly both contributions turn out to be equal to  $(-1/2)\mathbf{m}_0 \cdot \mathbf{B}_v(\mathbf{R}_d)$ .

#### *Interaction between a point magnetic dipole and the Meissner currents*

In the case of a superconductor without vortices [the so-called Meissner state,  $\text{rot}(\nabla\Theta) = 0$ ], the interaction between the dipole and the superconductor is given by  $U_m = (-1/2)\mathbf{m}_0 \cdot \mathbf{B}_m(\mathbf{R}_d)$ . It can easily be seen that this energy contribution is always positive, since the screening currents induced by a dipole generate a magnetic field which is opposite to the orientation of the dipole. This is in principle just a reformulation of the fact that a magnetic dipole will be repelled by a superconducting film with a force  $\mathbf{f} = -\partial U_m / \partial \mathbf{R}_d$ . The related problem of levitating ferromagnetic particle over various superconducting structures was considered theoretically by Wei *et al.* [147], Xu *et al.* [149], Coffey [150, 151] and Haley [152, 153]. In particular, in the limiting case  $Z_d/\lambda \gg 1$  the magnetic levitation force was found to vary linearly with  $\lambda$ , regardless of the shape of the magnet. The fact that  $\lambda$  (as well as the vertical component of the force) shows an exponential temperature dependence for  $s$ -wave superconductors, linear- $T$  for  $d$ -wave superconductors, and quadratic- $T^2$  dependence in a wide low-temperature range for materials with  $s+id$  symmetry of the gap, can assist in distinguishing the type of pairing in real samples based on magnetic force microscopy (MFM) measurements [149].

#### *Creation of vortices by the stray field of a magnetic dipole*

In reality, magnetic dipoles which have a high dipolar moment or are localized rather close to the superconductor can suppress the Meissner state due to the dipole's stray field. The possible appearance of a vortex would change the total energy of the S/F hybrid system by  $G_s = \epsilon_v^{(0)} + \epsilon_{mv}$ , where  $\epsilon_{mv} \propto -m_0$  accounts for the dipole-vortex interaction. Clearly, one can make  $G_s < 0$  by increasing  $m_0$  and therefore it will become energetically favorable to generate vortices in the system. Such a scenario for the formation of the mixed state was considered, e.g., by Wei *et al.* [147, 154] for a vertically magnetized dipole placed above a thin superconducting film. As a consequence of Eq. (20), the appearance of a vortex line in the superconductor drastically changes the resulting force acting on the magnetic dipole (Xu *et al.* [149], Wei *et al.* [147, 154]).

It is important to notice that in the case of superconducting films infinite in the lateral direction and cooled in zero magnetic field, a spontaneously induced vortex should be accompanied by an antivortex in order to provide flux conservation imposed by Maxwell's equations<sup>†</sup>. The destruction of the Meissner state in a zero-field-cooled superconducting film was theoretically studied by Melnikov *et al.* [155] and Aladyshkin *et al.* [156]. In this case the formation of the mixed state occurs via the penetration of vortex semi-loops which split into vortex-antivortex pairs. The local suppression of the Bean-Livingston energy barrier [157], which controls the process of vortex penetration, takes place when the screening current density will be of the order of the depairing current density. The threshold distance  $Z_d^* = Z_d^*(T)$ , corresponding to the suppression of the energy barrier can be experimentally detected by measuring a nonzero remanent magnetization as long as pinning is relevant. Interestingly, such non-contact technique allows one to estimate the depairing current density and its temperature dependence in thin superconducting  $\text{YBa}_2\text{Cu}_3\text{O}_7$  films [155, 156, 158]. Since the surface energy barrier in superconductors with a flat surface is known to be suppressed by applying an external field on the order of the critical thermodynamical field  $H_{cm} = \Phi_0/(2\sqrt{2}\pi\lambda\xi)$  [43], one can get a rough criterion for the persistence of the vortex-free state in the presence of a magnetic dipole:  $m_0/Z_d^3 \leq H_{cm}$ . A different criterion should be obtained in case the dipole is already present close to the superconductor and the whole system is cooled down below the superconducting temperature. Under this field-cooled condition a lower critical magnetization to induce a vortex-antivortex pair is expected.

The problem of the formation of vortex-antivortex pairs in superconducting films in the presence of vertically and horizontally magnetized dipole at  $H_{ext} = 0$  was considered theoretically by Milošević *et al.* [159, 160], Carneiro [148]. It was shown that an equilibrium vortex pattern could consist of spatially separated vortices and antivortices (for rather thin superconducting films) or, when the superconducting film thickness increases

<sup>†</sup> In mesoscopic superconducting systems, the returning flux lines generated by an out-of-plane magnetized dipole can bypass the border of the sample and the existence of an antivortex is not required.

and becomes comparable to the London penetration depth, curved vortex lines, which start and terminate at the surface of the superconductor.

### Magnetic pinning

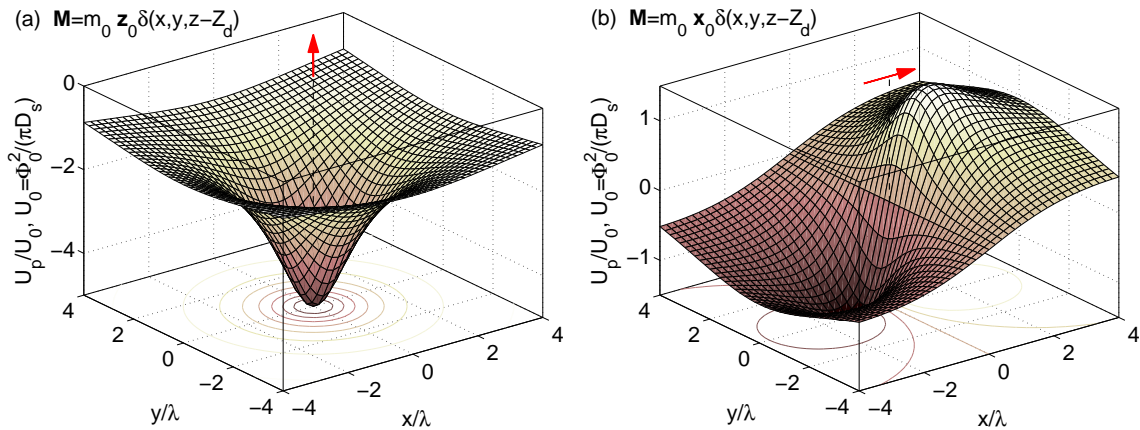
Now we will discuss the properties of S/F hybrids where the magnetization of the ferromagnet and the distance between ferromagnet and superconductor are assumed to be fixed resulting in a constancy of the interaction energy with the screening currents induced by the ferromagnet. In this case, any variation of the free energy of the S/F hybrid in the presence of vortices (either induced by the ferromagnetic element or by external sources) can be attributed to the rearrangements of the vortex pattern. The part of the interaction energy  $G_{int}$ , proportional to the magnetization of the ferromagnet and sensitive to the vortex' positions, is usually called the magnetic pinning energy  $U_p$ .

In order to illustrate the angular dependence of the interaction between a point magnetic dipole of fixed magnetization and a vortex, we refer to the following expression (Carneiro [148], Milošević *et al.* [159, 160])

$$U_p(r_\perp, 0) = \frac{\Phi_0^2}{2\pi\lambda} \left[ -\frac{1}{2} \left( \frac{m_{0,z}}{\Phi_0\lambda} \right) \frac{D_s}{(r_\perp^2 + Z_d^2)^{1/2}} + \frac{D_s}{2r_\perp} \left( \frac{m_{0,z}}{\Phi_0\lambda} \right) \left( 1 - \frac{Z_d}{(r_\perp^2 + Z_d^2)^{1/2}} \right) \cos \phi \right], \quad (21)$$

obtained for a thin superconducting film  $D_s \ll \lambda$  and for vortex-dipole separations smaller than the effective penetration depth  $\lambda_{2D} = \lambda^2/D_s$ . Here  $m_{0,x}$  and  $m_{0,z}$  are the in-plane and out-of-plane components of the dipolar moment,  $r_\perp = \sqrt{x^2 + y^2}$ ,  $\mathbf{R}_d = (0, 0, Z_d)$  is the position of the dipole,  $\phi$  is the angle between the  $x$ -axis and the vector position of the vortex in the plane of the superconductor  $\mathbf{r}_\perp$ . The resulting pinning potentials for an in-plane and out-of-plane magnetized dipole, derived from Eq. (21), are shown in Fig. 16. The pinning of vortices (and antivortices) in the superconducting films of a finite thickness on the magnetic dipole at  $H_{ext} = 0$  was analyzed by Milošević *et al.* [159, 160] and Carneiro [148, 161]. In addition, Carneiro considered the magnetic pinning for the case of a dipole able to rotate freely in the presence of the external magnetic field [162, 163, 164] or external current [162].

In Ref. [148] it was shown that this magnetic pinning potential has a depth of  $m_0/(4\pi\lambda)$ , penetrates a distance  $\lambda$  into the film whereas its range parallel to the film surface is a few times  $\lambda$ . This finding points out the relevance of the penetration depth  $\lambda$  to characterize the purely magnetic pinning potential. Since typical  $\lambda$  values are similar for a broad spectrum of superconducting materials, this suggests that magnetic pinning represents a promising way of increasing the critical current not only in conventional superconductors but also in high- $T_c$  superconductors. Notice that the fact that the magnetic pinning range is determined by  $\lambda$  sets a limit for the minimum distance between magnetic particles, beyond which the vortex lines cannot resolve the field modulation and therefore the pinning efficiency decreases. Having in mind that many practical applications



**Figure 16.** (color online) The spatial dependence of the energy of a single vortex in a field of point magnetic dipole,  $\mathbf{M} = \mathbf{m}_0 \delta(x, y, z - Z_d)$ , calculated according to Eq. (21) for the height  $Z_d/\lambda = 1$  and the dipole strength  $m_0/(\Phi_0\lambda) = 10$ :

- (a) vertically magnetized dipole,  $\mathbf{m}_0 = m_0 \mathbf{z}_0$ ,
- (b) horizontally magnetized dipole,  $\mathbf{m}_0 = m_0 \mathbf{x}_0$ .

typically involve high- $T_c$  superconductors with  $\xi \ll \lambda$ , it becomes clear that this maximum density of pinning sites to trap individual vortices is much lower than that limited due to core pinning.

It is important to mention that real nanoengineered ferromagnetic elements are quite far from point dipoles but rather consist of extended volumes of a magnetic material, such as magnetic dots and stripes. In this case, due to the principle of superposition, the magnetic pinning energy needs to be integrated over the volume of the ferromagnet  $V_f$  to determine the interaction between a vortex line and a finite size permanent magnet

$$U_p(\mathbf{r}) = - \int_{V_f} \mathbf{M}(\mathbf{r}') \cdot \mathbf{B}_v(\mathbf{r} - \mathbf{r}') d^3r'. \quad (22)$$

The magnetic pinning in an infinite superconducting film produced by individual ferromagnetic objects of finite size was considered for the following cases: a ferromagnetic sphere magnetized out-of-plane (Tokman [165]), magnetic disks, rings, rectangles, and triangles magnetized out-of-plane (Milošević *et al.* [160, 166]), magnetic bars and rectangular dots magnetized in-plane (Milošević *et al.* [167]), circular dots magnetized either in-plane and out-of-plane (Erdin *et al.* [146], Erdin [168]), circular and elliptic dots and rings magnetized out-of-plane (Kayali [169, 170], Helseth [171]), columnar ferromagnetic rod (Kayali [172]). The analysis of the pinning properties of a vortex in a semi-infinite superconducting film due to magnetic dots was done by Erdin [173]. Similar to the case of a point magnetic dipole, the extended magnetic objects are able to generate vortex-antivortex pairs provided that the size and the magnetization are large enough.

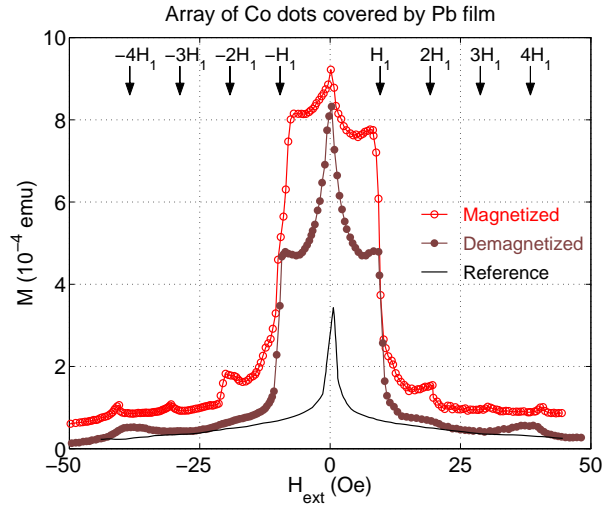
We would like to note that the attraction of a superconducting vortex to a source of inhomogeneous magnetic field (e.g., coil on a superconducting quantum interference device or magnetized tip of a magnetic force microscope) makes it possible to precisely manipulate the position of an individual vortex. Such experiments were performed by Moser *et al.* [174], Gardner *et al.* [175], Auslaender *et al.* [176] for high- $T_c$  superconducting thin films and single crystals in intermediate temperatures when intrinsic pinning become weaker. This technique apparently opens unprecedented opportunities, e.g., for a direct measuring of the interaction of a moving vortex with the local disorder potential and for preparing exotic vortex states like entangled vortices (Reichhardt [177]).

Equation (22) shows that the pinning potential does not only depend on the size and the shape of the ferromagnetic elements but also on the particular distribution of the magnetization (i.e. on their exact magnetic state). It is expected that the average pinning energy is less efficient for magnetic dots in a multidomain state whereas it should reach a maximum for single domain structures. In other words, if the size of the magnetic domains is small in comparison with  $\lambda$  or the separation distance  $Z_d$ , then a vortex line would feel the average field emanating from the domains and the magnetic pinning should be strongly suppressed. This flexibility of magnetic pinning centers makes it possible to tune the effective pinning strength, as we shall discuss below.

The question now arises whether the pinning potential produced by a magnetic dipole will remain efficient when a vortex-antivortex pair is induced by the magnetic dipole. In order to answer this question it is necessary to minimize the mutual interaction between the induced vortex-antivortex pair, the magnetic element and the test vortex generated by the external field. The magnetic moment – test vortex interaction is a linear function of  $m_0$  always favoring the test vortex to sit on the positive pole of the magnet, whereas the induced currents – test vortex interaction will follow the Little-Parks oscillations (due to the creation of vortices by the dipole). For the case of an in-plane dipole, Milošević *et al.* [167] demonstrated that all these terms produce a subtle balance of forces which lead to a switching of the optimum pinning site from the positive to the negative magnetic pole, as  $m_0$  is increased. For the case of out-of-plane magnetic dipole, once a vortex-antivortex pair is induced, the test vortex will annihilate the antivortex leaving a single vortex on top of the magnetic dot (Gillijns *et al.* [123]).

### 3.3. Magnetic dots in the vicinity of a plain superconducting film

The rich variety of possibilities considered theoretically in the previous section for an individual ferromagnetic element in close proximity to a superconducting film represents an experimental challenge in part due to the difficulties associated with recording small induced signals. A very successful way to overcome this limitation consists of studying the average effect of a periodic array of dots at the expense of blurring sharp transitions, such as the vortex-antivortex generation, or inducing new collective phenomena associated with the periodicity of the magnetic templates. An important experimental condition that should be satisfied in order to justify the analogies drawn between individual dipoles and arrays of dipoles, is the lack of magnetostatic interactions



**Figure 17.** (color online) Upper half of the magnetization loop  $M$  vs.  $H_{ext}$  at  $T/T_{c0} = 0.97$ , for a superconducting Pb film (50 nm thickness) on top of a triangular lattice of Au/Co/Au dots (period  $1.5 \mu\text{m}$  corresponding to a first matching field of 9.6 Oe) before and after magnetizing the dots (filled and open symbols, respectively), and for a reference Pb 50 film (solid line), adapted from van Bael *et al.* [196]. The curve for the magnetized dots is slightly shifted upwards for clarity.

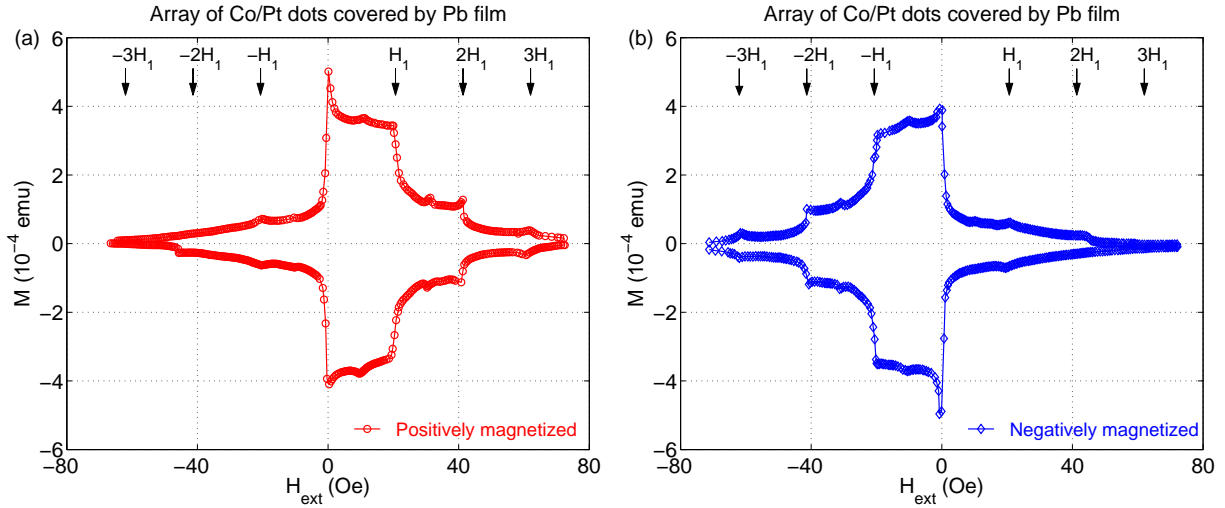
between neighboring dipoles. In other words, it is necessary to ensure that the field generated by a dipole at the position of its nearest neighbors lies below the coercive field of the chosen ferromagnetic materials (Cowburn *et al.* [178, 179], Novosad *et al.* [180, 181]).

#### *Commensurability effects in S/F hybrids with periodic arrays of magnetic dots*

In the early 1970's, Autler [182, 183] proposed that a periodic array of ferromagnetic particles should give rise to an enhancement of the critical current of the superconducting material. Recent developments on lithographic techniques have made it possible to prepare superconducting structures containing a regular array of magnetic dots (Co, Ni, Fe) at submicrometer scale of desirable symmetry in a controlled way (Otani *et al.* [119], Geoffroy *et al.* [120], Martín *et al.* [184, 185, 186, 187], Morgan and Ketterson [188, 189], Hoffmann *et al.* [190], Jaccard *et al.* [191], Villegas *et al.* [192, 193, 194], Stoll *et al.* [195], van Bael *et al.* [196, 197, 198, 199, 200, 201], van Look *et al.* [202]). The S/F hybrids containing periodic arrays of magnetic elements with out-of-plane magnetization (multilayered Co/Pt and Co/Pd structures) were fabricated and investigated by van Bael *et al.*, [198, 199, 200, 201, 203, 204] and Lange *et al.* [205, 206]. In all these works, it was found that the presence of the lattice of magnetic dots leads either to (i) a resonant change in the magnetoresistance  $\rho(H_{ext})$  and the appearance of pronounced equidistant minima of resistivity with the period  $H_1 = \Phi_0/S_0$  determined by the size of the magnetic unit cell  $S_0$  [119, 120, 184, 185, 186, 187, 189, 190, 191, 193, 192, 194, 195]; (ii) or to the presence of distinct features as peaks or plateaus in the field dependence of the critical current  $I_c(H_{ext})$  or in the magnetization curve  $M(H_{ext})$  [188, 189, 190, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206].

Such matching effects are also very well known for spatially-modulated superconducting systems with antidot lattices without ferromagnetic constituents (e.g., Baert *et al.* [52], Moshchalkov *et al.* [53], Hebard *et al.* [207], Rosseel *et al.* [208], Harada *et al.* [209], Metlushko *et al.* [210]). These periodic anomalies are commonly explained in terms of commensurability effects between the vortex lattice governed by the external field and the artificially-introduced pinning potential at an integer number of vortices per unit cell at  $H_{ext} = \pm nH_1$  (with  $n$  integer). Typically these lithographically defined arrays are limited to a minimal period of the unit cell of the order of a few hundred nanometers, giving rise to a maximum matching field  $H_1 \simeq 10 - 10^2$  Oe. Alternative methods for introducing more closely packed particles and thus larger  $H_1$  values can be achieved by Bitter decoration (Fasano *et al.* [211, 212], Fasano and Menghini [213]) or a diversity of self-assembled techniques (Goyal *et al.* [214], Villegas *et al.* [215], Welp *et al.* [216, 217], Vinckx *et al.* [218], Vanacken *et al.* [219]).

The field dependence of the critical current  $I_c(H_{ext})$  and magnetization  $M(H_{ext})$ , can be symmetrical or asymmetrical with respect to  $H_{ext} = 0$  depending on the dot's net magnetization (compare Figs. 17 and 18).

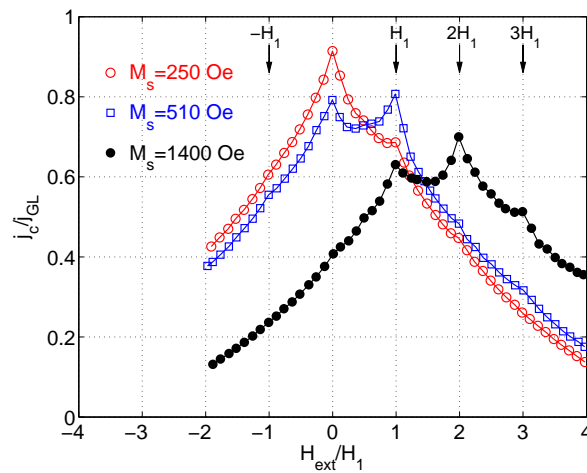


**Figure 18.** (color online) Magnetization curves  $M$  vs.  $H_{ext}$  at  $T = 7$  K ( $T_{c0} = 7.17$  K,  $T/T_{c0} = 0.976$ ) for a superconducting Pb film (50 nm thickness) on top of a Co/Pt dot array (the period is  $1.0 \mu\text{m}$ , the first matching field is 20.68 Oe) with all dots aligned in the positive (a) and negative (b) direction, adapted from van Bael *et al.* [204].

The latter takes place for arrays of magnetic dots with out-of-plane magnetization [198, 199, 200, 201, 203, 204, 205, 206]. Indeed, vertically magnetized dots with average magnetic moment  $\langle \mathbf{m} \rangle_z > 0$ , similar to point magnetic dipoles, produce a stronger pinning potential for vortices (at  $H_{ext} > 0$  when  $\langle \mathbf{m} \rangle \parallel \mathbf{H}_{ext}$ ) than for antivortices ( $H_{ext} < 0$ ). In contrast to that, in-plane magnetized dots are able to pin vortices and antivortices at the magnetic poles equally well [see Eq. (21) and Fig. 16]. This explains the experimentally observed field polarity-dependent (asymmetric) pinning for arrays of out-of-plane magnetized particles (Fig. 18 and Fig. 19).

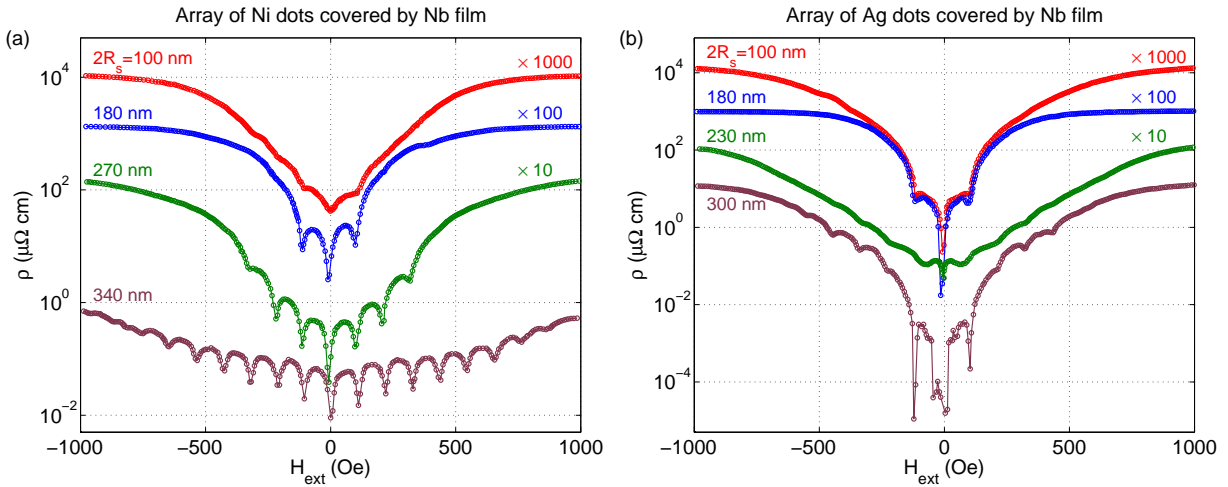
The interaction between vortices and a periodic array of hard magnetic dots on top or underneath a plain superconducting film within the London approximation<sup>†</sup> was theoretically analyzed by Helseth [225], Lyuksyutov

<sup>†</sup> This issue seems to be part of a more general problem of the interaction of vortex matter with a periodic potential regardless the nature of the pinning in the superconducting system (see, e.g., Reichhardt *et al.* [220, 221, 222, 223, 224] and references therein). In this review we discuss only the results obtained for the S/F hybrids, keeping in mind that similar effects can be observed for



**Figure 19.** (color online) Field dependence of the critical depinning current  $j_c$ , calculated for a periodic array of out-of-plane magnetized dots. The values of the dot's magnetization are indicated in the plot.  $j_{GL}$  is the density of the depairing (Ginzburg-Landau) current,  $T/T_{c0} = 0.9$ , adapted from Milošević and Peeters [122]. It is worth noting that (i)  $j_c(H_{ext})$  is asymmetric similar to that shown in Fig. 18, and (ii) the quantized displacement of the  $j_c$  maximum toward nonzero  $H_{ext}$  value is sensitive to the magnetization of the dots.





**Figure 20.** (color online) (a) Field dependence of the electrical resistance  $\rho$  for a superconducting Nb films covering periodic arrays of the magnetic Ni dots with different dot's diameter (indicated in the plot), but for the same lattice constant 400 nm, adapted from Hoffmann *et al.* [190]. The curves are shifted by factors of 10 from each other for clarity.

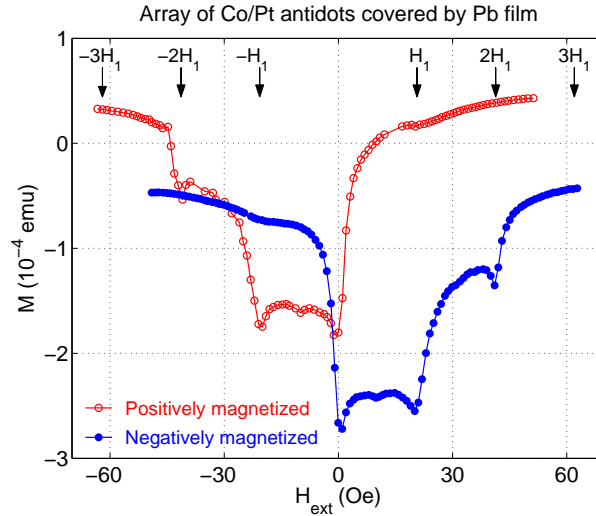
(b) Dependences  $\rho(H_{\text{ext}})$  for samples with different diameters of non-magnetic Ag dots, adapted from Hoffmann *et al.* [190].

and Pokrovsky [226], Šášik and Hwa [227], Erdin [228], Wei [229, 230], Chen *et al.* [231]. These calculations show that at  $H_{\text{ext}} = 0$  for out-of-plane magnetized dots, vortex-antivortex pairs can be created in thin-film superconductors with the vortices always sitting on top of the magnetic dot and the antivortices located in between the dots. For in-plane magnetized dots (or magnetic bars), the vortex and antivortex will be located at opposite sides of the magnetic dots as described above for individual magnetic dipoles. Unlike the case of an isolated dipole, the threshold magnetization value needed to create a vortex-antivortex pair is also a function of the period of the lattice (Milošević and Peeters [114]). Direct visualization of vortex-antivortex pairs via scanning Hall probe microscopy was achieved for a square array of in-plane dots by van Bael *et al.* [197] and for out-of-plane dots by van Bael *et al.* [204] and Neal *et al.* [232].

It is known that the preferred vortex configuration in a homogeneous defect-free superconducting film should be close to a triangular lattice because of the repulsive vortex-vortex interaction [43, 44, 45]. The artificially-introduced pinning appears to be the most effective provided that each vortex is trapped by a pinning center, i.e. when the symmetry of the pinned vortex lattice coincides with that imposed by the topology of the internal magnetic field. The transition between square and distorted triangular vortex lattice, induced by variation of the strength of the periodic pinning potential and the characteristic length scale of this interaction, was considered by Pogosov *et al.* [233] for superconductors with a square array of pinning centers. Experimentally the field-induced reconfiguration of the vortex lattice (from rectangular to square) for superconducting Nb films and rectangular arrays of circular magnetic Ni and Co dots was reported by Martín *et al.* [185] and Stoll *et al.* [195] as an abrupt increase of the period of the oscillation in the  $\rho(H_{\text{ext}})$  dependence (resulting from the shrinkage of the period of the vortex lattice) and decrease of the amplitude of such oscillations (due to a weakening of the effective pinning) while increasing  $|H_{\text{ext}}|$ .

The dependence of the magnetic pinning in superconducting Nb films on the diameter of the Ni dots was studied by Hoffmann *et al.* [190]. They found that more minima appear in the magnetoresistance (or maxima in the critical current) as the lateral dot's size increases, indicating thus an enhanced pinning (the panel (a) in Fig. 20). This effect can be caused by the two parameters which increase with the dot size: the total magnetic moment  $\langle \mathbf{m} \rangle$  (proportional to the dot's volume  $V_f = \pi R_f^2 D_f$ ) and the area on the order of  $\pi R_f^2$  where superconductivity might be locally suppressed due to the high stray field or proximity effect. In addition, larger magnetic dots can stabilize giant vortices carrying more than one flux quantum.

non-magnetic patterned superconductors as well.



**Figure 21.** (color online) Magnetization curves  $M(H_{ext})$  at  $T = 7.05$  K ( $T_{c0} = 7.20$  K,  $T/T_{c0} = 0.972$ ) of a superconducting Pb film on top of a magnetic Co/Pt antidot lattice (the period is  $1.0 \mu\text{m}$ , the first matching field is  $20.68$  Oe) after saturation in a positive field ( $M_z > 0$ , open circles) and after saturation in a negative field ( $M_z < 0$ , filled circles), adapted from Lange *et al.* [234, 235].

#### Periodic arrays of magnetic antidots

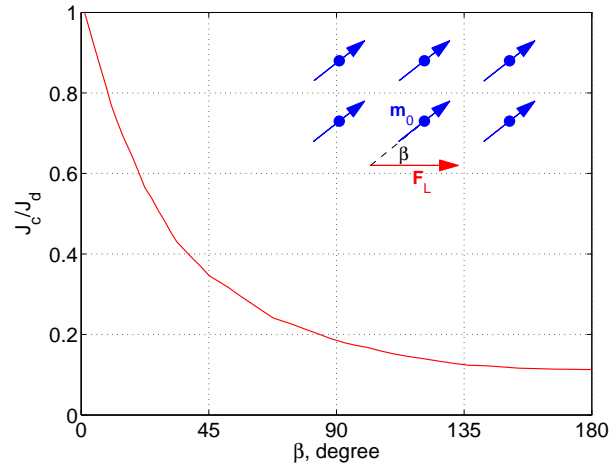
The antipode of arrays of magnetic dots is a perforated ferromagnetic film (so-called magnetic antidots), which also produces a periodic magnetic field. This system can be regarded as the limiting case of big magnetic dots with a diameter larger than the period of the periodic lattice.

Magnetic antidots in multilayered Co/Pt films, characterized by an out-of-plane remanent magnetization, and their influence on the superconducting properties of Pb films were studied by Lange *et al.* [205, 234, 235, 236]. From magnetostatic considerations, such submicron holes in a ferromagnetic thin film generates a very similar field pattern as an array of magnetic dots of the same geometry, but with opposite sign. As a consequence, the enhanced magnetic pinning and the pronounced commensurability peaks in the  $M(H_{ext})$  dependence are observed for the opposite polarity of the external field (i.e. at  $H_{ext} < 0$  for positively magnetized film and vice versa), see Fig. 21. However the matching effects are considerably weakened in the demagnetized state of the Co/Pt multilayer with holes as compared with the demagnetized array of magnetic dots, thus indicating that an irregular domain structure effectively destroys a long-range periodicity [205].

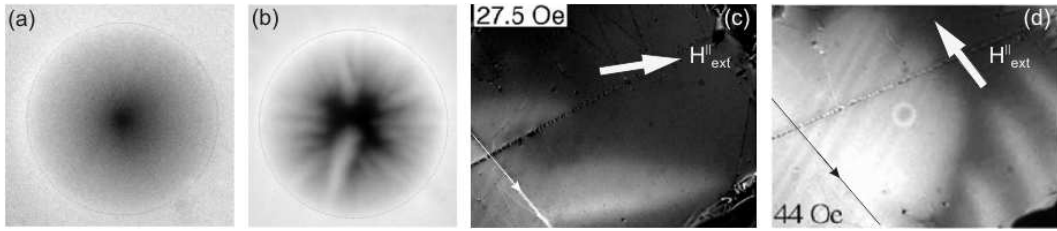
Van Bael *et al.* [237] and Raedts *et al.* [238] explored perforated Co film with in-plane anisotropy. In this case the magnetic field distribution becomes non-trivial since such magnetic antidots effectively pin magnetic domain walls which generate a rather strong magnetic field. As a result, neither matching effects nor pronounced asymmetry were observed in the magnetization curves of the superconducting layer, but only an overall enhancement of the critical current after the sample was magnetized along the in-plane easy axis, in comparison with the demagnetized state.

#### Anisotropy of the transport characteristics and guidance of vortices

In any periodic array of pinning centra, transport properties such as magnetoresistance  $\rho(H)$  and the critical current  $J_c(H)$ , exhibit a dependence not only on the absolute value of  $H_{ext}$ , but also on the direction of the applied transport current with respect to the principal translation vectors of the periodic pinning array (Villegas *et al.* [193, 194], Soroka and Huth [239], Vélez *et al.* [240], Silhanek *et al.* [241], Wördenweber *et al.* [242]). Interestingly, the direction of the Lorentz force  $\mathbf{f}_L = c^{-1}[\mathbf{j} \times \mathbf{B}]$  and the drift velocity of the vortex lattice do not generally coincide. It was demonstrated that for rectangular arrays of magnetic dots the minimum of resistivity corresponds to a motion of the vortex lattice along the long side of the array cell. Such behavior was predicted by Reichhardt *et al.* [223] by numerical simulations indicating that a rectangular array of pinning centers



**Figure 22.** (color online) Critical current,  $J_c$ , for a vortex pinned by a dipole array as a function of the angle  $\beta$  between the magnetic moments and the driving force, adapted from Carneiro [243]. Here  $J_d$  is the depairing current.



**Figure 23.** (a) Magneto-optical image of a non-patterned superconducting Pb disk at  $T = 2$  K and  $H_{ext} = 50$  Oe, demonstrating an isotropic flux penetration, after Gheorghe *et al.* [246]. White corresponds to a high local magnetic field and black to zero local field. (b) Magneto-optical image of a circular Pb sample decorated by fully magnetized Co/ Pt dots, obtained at  $T = 2$  K and  $H_{ext} = 72$  Oe, after Gheorghe *et al.* [246]. The external field is applied parallel to the dot's magnetic moment. (c) Magneto-optical image of flux entry in a superconducting MoGe film at  $H_{ext}^{\parallel} = 16.5$  Oe,  $T = 4.5$  K following the preparation of stripe domain structures in a permalloy film by turning on and off an in-plane field of  $H_{ext}^{\parallel} = 1$  kOe at an angle of  $45^\circ$  with respect to the sample edge at  $T > T_{c0}$ , after Vlasko-Vlasov *et al.* [247]. The brightness of the magneto-optical contrast corresponds to the vortex density. The large yellow arrow shows the preferential flux entry direction coinciding with the direction of the stripe domains in the Py film. The thin solid line with arrow marks the sample edge. (d) Same as in panel (c) after application and switching off of  $H_{ext}^{\parallel} = 1$  kOe along the sample edge at  $T > T_{c0}$ , after Vlasko-Vlasov *et al.* [247].

induces an easy direction of motion for the vortex lattice (and larger dissipation as well) along the short side of the array cell.

Similar anisotropic transport properties was studied by Carneiro [243] for the case of a periodic array of in-plane magnetic dipoles. In order to illustrate the angular dependence of the critical depinning current on the angle  $\beta$  between the direction of the injected current and the magnetic moment of in-plane oriented dipoles we refer to Fig. 22. Interestingly, Verellen *et al.* [244] showed that this resulting guided vortex motion in square arrays of magnetic rings can be rerouted by  $90^\circ$  simply by changing the dipole orientation or can even be suppressed by inducing a flux-closure magnetic vortex state with very low stray fields in the rings. Similar anisotropic vortex motion was recently observed in Nb films with a periodic array of one-dimensional Ni lines underneath by Jaque *et al.* [245]. The mentioned channelling of vortices lead to an anisotropic vortex penetration that has been directly visualized by means of magnetooptics experiments [Gheorghe *et al.* [246], see Fig. 23(b)].

#### *Mechanisms of pinning in S/F hybrids*

It should be noticed that the magnetic pinning originating from the spatial modulation of the “internal” magnetic

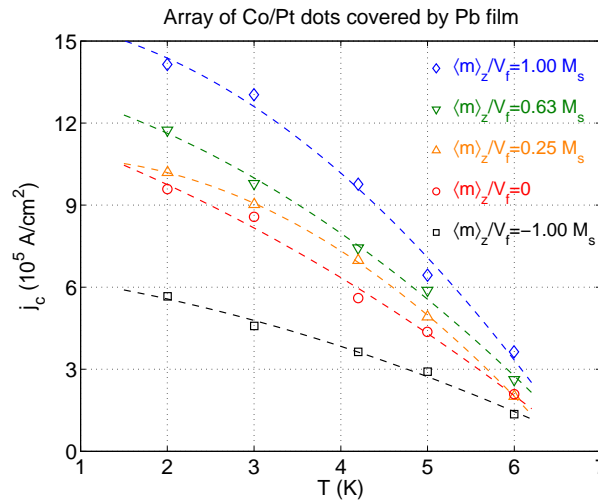
field generally competes with so-called core pinning resulting from structural inhomogeneities in real samples (either regular or random defects). In addition to random intrinsic pinning, the fabrication of an array of magnetic particles naturally leads to an alteration of the local properties of the superconducting film (e.g., due to proximity effects, corrugation of the superconducting layer or local suppression of the critical temperature). As a consequence, both magnetic and structural modulation share the same periodicity, and a clear identification of the actual pinning type becomes difficult.

A direct comparison of the pinning properties of arrays of magnetic vs. nonmagnetic dots have been addressed by Hoffmann *et al.* [190] and Jaccard *et al.* [191]. These reports show that even though both systems display commensurability features, the pinning produced by magnetic arrays of Ni dots is substantially stronger than that produced by non-magnetic Ag particles (Fig. 20). In our opinion, the main issue whether the enhanced pinning for the sample with ferromagnetic Ni dots actually arises from purely magnetic interactions and not from an additional suppression of the local critical temperature, e.g. due to the enhanced magnetic field near magnetic dots, remains unclear. In principle, the most straightforward way to distinguish the two competing pinning mechanisms is the mentioned field-polarity of the magnetic pinning for the S/F hybrids with dots magnetized perpendicularly, i.e. exploring the broken field-polarity symmetry.

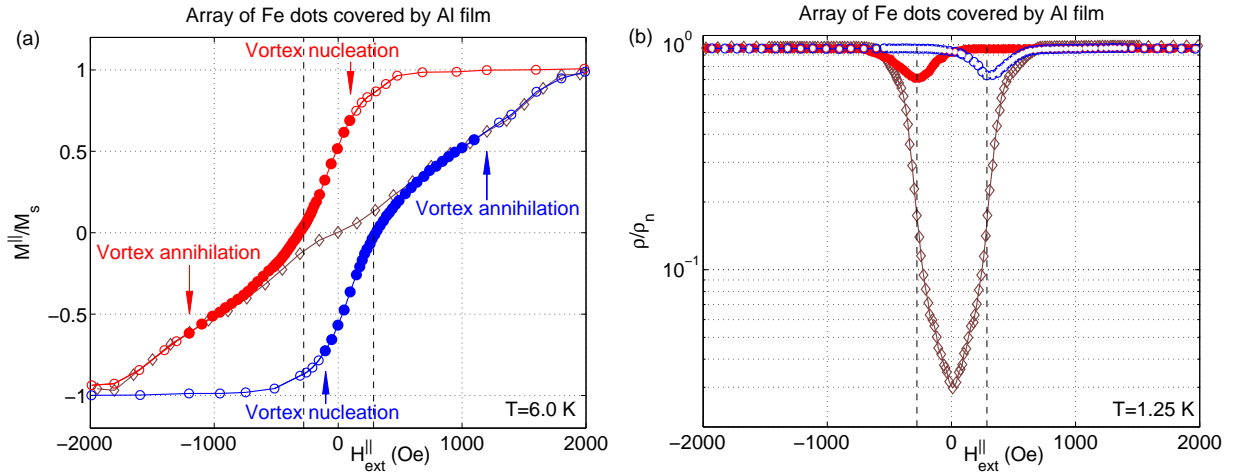
Clear evidence of the field-polarity dependent pinning properties has been reported by Gheorghe *et al.* [246] in Pb films on top of a square array of [Co/Pt]<sub>10</sub> dots with a well defined out-of-plane magnetic moment. In this work the authors show that the critical current of the hybrid system can be increased by a factor of 2 when the magnetic dots are switched from low stray field in the demagnetized state (disordered magnetic moment) to high stray field in the magnetized state (nearly single domain state) at temperatures as low as  $T \simeq 0.3 T_{c0}$  (see Fig. 24). Additional evidence of an increase of the critical current at low temperatures (far from the superconducting/normal phase boundary) produced by magnetic dots was reported by Terentiev *et al.* [248, 249, 250].

#### Tunable pinning centers

An apparent advantage of using magnetic pinning centra is their flexibility (tunability) in contrast to core pinning on structural inhomogeneities. Indeed, according to Eq. (22) the magnetic pinning should be sensitive to the particular distribution of magnetization inside the ferromagnetic elements. Depending on the geometrical details of the dot and the magnetic anisotropy of the chosen material a huge variety of magnetic states can be found. For instance, domain formation is expected to be suppressed for structures with lateral dimensions



**Figure 24.** (color online) Temperature dependence of the critical current density  $j_c$ , estimated from magneto-optical images, for a superconducting Pb film with square array of the ferromagnetic Co/Pt dots on top, in various magnetic states of the dots: demagnetized ( $\circ$ ), fully magnetized parallel configuration ( $\diamond$ ), fully magnetized antiparallel configuration (square), partially magnetized parallel,  $\langle m \rangle_z = 0.25 M_s V_f$  ( $\Delta$ ) and  $\langle m \rangle_z = 0.63 M_s V_f$  ( $\nabla$ ), adapted from Gheorghe *et al.* [246]. The dashed lines are guides to the eye.

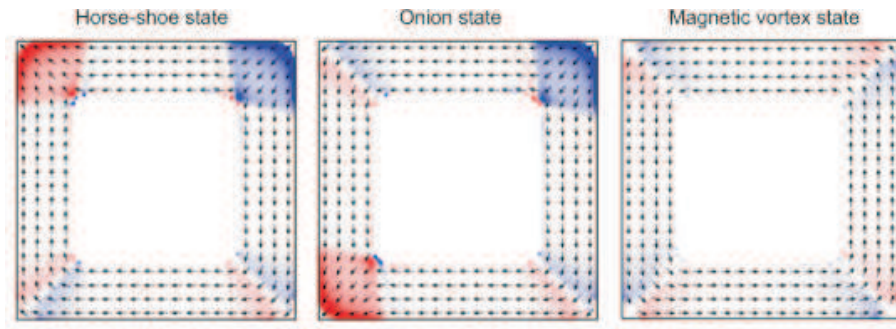


**Figure 25.** (color online) (a) Normalized magnetization  $M^{\parallel}/M_s$  vs. in-plane applied field  $H_{ext}^{\parallel}$  ( $M_s$  is the saturated magnetization) for an array of Fe dots with average diameter of 140 nm and average interdot distance of 180 nm measured at  $T = 6$  K (above the critical temperature of superconducting Al film), adapted from Villegas *et al.* [215]. Brown diamonds correspond to a virgin state of the Fe dots, the magnetic state depicted by red (blue) circles obtained after saturation in positive (descending branch) and negative (ascending branch) magnetic fields, respectively. The filled circles schematically show the regions where the magnetic vortex is expected to take place. Vertical dashed lines mark the coercive fields. (b) Normalized resistivity  $\rho$  vs. in-plane applied field  $H_{ext}^{\parallel}$  for the same sample at  $T = 1.25$  K (below the critical temperature of superconducting Al film),  $\rho_n$  is the normal-state resistance. Open (blue) and filled (red) circles mark the curves measured from negative and positive saturation respectively, while brown diamonds correspond to the virgin state, adapted from Villegas *et al.* [215].

smaller than tens of nm (Raabe *et al.* [251]), whereas for larger sizes the magnetic sample breaks into domains of different orientation (Seynaeve *et al.* [252]). The exact transition from single domain to multi-domain structures depends on the shape, dimensions, temperature and particular material, among other parameters. More recently, Villegas *et al.* [215, 253] and Hoffmann *et al.* [254] experimentally investigated the switching of the ferromagnetic dots from single domain to magnetic vortex state while sweeping the external field and the influence of their stray fields on the resistivity of the S/F hybrid sample (Fig. 25). The interaction between a vortex in a superconducting film and a magnetic nanodisk in the magnetic vortex state was studied theoretically by Carneiro [255]. For magnetic dots big enough to host a multi-domain state it is possible to tune the average magnetic moment by partially magnetizing the sample in a field lower than the saturation field or even recover the virgin state by performing a careful degaussing procedure similar to that shown in Fig. 6 (Gillijns *et al.* [96, 123], Lange *et al.* [256]). Interestingly, recently Cowburn *et al.* [178] showed that small disks of radius about 50 nm made of supermalloy (Ni-80%, Fe-14%, Mo-5%) lie in a single domain state with the magnetization parallel to the disk plane and with the property that their direction can be reoriented by small applied fields. This systems represents the closest experimental realization of in-plane free-rotating dipoles, which was theoretically analyzed by Carneiro [162, 163, 164] within the London formalism.

Whatever the mechanism of pinning produced by the magnetic dots, either core or electromagnetic, it is now a clearly established fact that changing the domain distribution in each dot has profound effects on the superconducting pinning properties as demonstrated, for example, by van Bael *et al.* [196], van Look *et al.* [202]. This result points out the importance of performing a careful study of the magnetic properties of the dots in order to identify the domain size, shape, distribution, and stable states. Van Bael *et al.* [196] presented the first report directly linking changes in the hysteresis loop of a superconducting Pb film when the underlying submicron Co islands are switched from  $2 \times 2$  checkerboard magnetic domain pattern to single domain structures.

As we pointed out above, unfortunately both, the multidomain state and the magnetic vortex state, still involve a sizable component of the magnetic stray field which eventually influences the response of the superconducting properties by locally suppressing the order parameter. In other words, it is actually not



**Figure 26.** (color online) Different magnetic states realized in square permalloy micro-loops depending on the direction of the applied magnetic field, by courtesy of Metlushko *et al.*

possible to completely switch off the magnetic pinning using singly connected structures. It has been recently shown that a way to partially circumvent this limitation can be achieved by using multiply connected ring-like magnetic structures. In this case, if a flux-closure state is induced in the magnetic ring, in principle, there is almost no stray field present, besides small fields due to domain walls at the sharp corners of the ring. Indeed, a two-dimensional magnetic material of ring like shape of group symmetry  $C_n$  can be set in two flux closure states of opposite chirality and  $n(n-1)$  different polarized states. In a square loop, for instance, 12 states corresponding to six different dipole directions with two opposite dipole orientations are expected. If the net dipolar moment is parallel to the side of the square, the final state is named horseshoe state whereas if the dipole is along the diagonal of the square, it is called onion state. Figure 26 shows the different topologically non-equivalent magnetic states for a square ring of magnetic material with in-plane magnetization. Clear experimental evidence of ON/OFF magnetic pinning potentials induced by these type of multiply-connected ferromagnetic structures have been demonstrated by Silhanek *et al.* [257, 258, 259]. It is worth mentioning that the S/F structures investigated in Ref. [258] exhibit two well distinguished phases corresponding to a disordered phase when the sample is in the as-grown state and an ordered phase when the sample is magnetized with an in-plane field. These order-disorder transitions manifest themselves as an enhancement of submatching features in the field dependence of the critical current which cannot be explained from a simple rescaling of the response corresponding to the disordered phase.

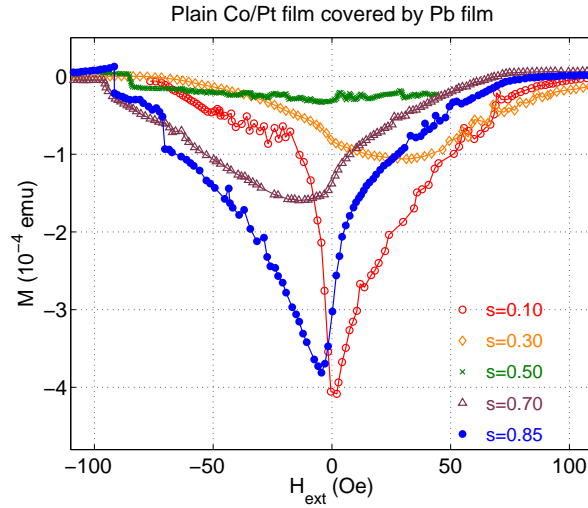
#### *Random (disordered) magnetic inclusions*

Early studies of the influence of ferromagnet on the superconducting state were performed in the sixties by Strongin *et al.* [260], Alden and Livingston [261, 262] and Koch and Love [263] for a dispersion of fine ferromagnetic particles (Fe, Gd, Y) in a superconducting matrix. These reports motivated further experimental and theoretical investigations of the influence of randomly distributed particles on/underneath/inside superconducting materials, which continue nowadays (Sikora and Makiej [264, 265], Wang *et al.* [266], Lyuksyutov and Naugle [267, 268, 269], Santos *et al.* [270], Kuroda *et al.* [271], Togoulev *et al.* [272], Kruchinin *et al.* [273], Palau *et al.* [274, 275], Haindl *et al.* [276, 277], Snezhko *et al.* [278], Rizzo *et al.* [279], Stamopoulos *et al.* [280, 281, 282, 283, 284], Suleimanov *et al.* [285], Xing *et al.* [286, 287, 288]). In most of these investigations no precautions were taken to electrically isolate the magnetic particles from the superconducting material which presumably results in a substantial core pinning due to proximity effects.

Xing *et al.* [287] reported on controlled switching between paramagnetic<sup>†</sup> and diamagnetic Meissner effect in S/F nanocomposites consisting of Pb films with embedded single-domain Co particles. These authors argue that in this particular system the paramagnetic Meissner effect attributed to the superconducting part only, originates from the spontaneous formation of vortices induced by the ferromagnetic inclusions. Therefore, the different contributions of the external field and the spontaneous vortices to the resulting magnetization of the sample, make it possible to manipulate the sign of Meissner effect by changing the orientation of the magnetic

<sup>†</sup> The paramagnetic Meissner effect in various superconducting systems is discussed in the review of Li [289].





**Figure 27.** (color online) Bottom parts of the magnetization curves  $M$  vs.  $H_{ext}$  for a superconducting Pb film covering a Co/Pt multilayer. The curves  $M(H_{ext})$  corresponding to the different values of the parameters  $s$ , which is defined as the fraction of magnetic moments that are pointing up ( $m > 0$ ) relative to the total number of magnetic moments:  $s = 0.1$  (open circles),  $s = 0.3$  (diamonds),  $s = 0.5$  (crosses),  $s = 0.7$  (triangles) and  $s = 0.85$  (filled circles), adapted from Lange *et al.* [256].

moments embedded in the superconducting matrix<sup>‡</sup>.

#### *Vortex dynamics in a periodic magnetic field*

Here we want to briefly discuss the peculiarities of low-frequency vortex dynamics in nonuniform magnetic fields. Magnetic templates placed in the vicinity of a superconducting film not only induces changes in the static pinning properties but also in the overall dynamic response of the system. Lange *et al.* [206] demonstrated that the vortex-antivortex pairs induced by an array of out-of-plane magnetized dots lead to a strong field polarity dependent vortex creep as evidenced in the current-voltage characteristics. This result shows that in S/F hybrids with perpendicular magnetized dots vortices and antivortices experience a different pinning strength. A theoretical study of the dynamic evolution of these interleaved lattices of vortices and antivortices in the case of in-plane point like dipoles has been recently addressed by Carneiro [291] and Lima and de Souza Silva [292].

A more subtle effect, namely magnetic-dipole-induced voltage rectification was predicted by Carneiro [243]. Unlike conventional ratchet systems<sup>§</sup>, in the particular case of magnetic ratchet, induced by in-plane magnetized dots, the motion of vortices is in the opposite direction than the motion of antivortices, thus giving rise to a field-polarity independent rectification (Silhanek *et al.* [258, 259], de Souza Silva *et al.* [303]). This magnetic dipole-induced ratchet motion depends on the mutual orientation and strength of the local magnetic moments thus allowing one to control the direction of the vortex drift. In some cases, a nonzero rectified signal is observed even at  $H_{ext} = 0$  resulting from the interaction between the induced vortex-antivortex pairs by the magnetic dipoles [303]. It is worth emphasizing that in the case of in-plane magnetic dipoles treated by Carneiro [243], the inversion symmetry is broken by the stray field of the dipoles, thus giving rise to different depinning forces parallel and anti-parallel to the dipoles orientations, as shown in Fig. 22.

<sup>‡</sup> Previously, Monton *et al.* [290] reported on an experimental observation of the paramagnetic Meissner effect in Nb/Co superlattices in field-cooled measurements, however the origin of this effect remains unclarified.

<sup>§</sup> Early theoretical studies showed that a vortex lattice submitted to an oscillatory excitation in the presence of a non-centro-symmetric pinning potential gives rise to a net drift  $\langle \mathbf{v} \rangle$  of the vortex lattice which in turn generates a dc voltage signal  $V_{dc} = \int [(\langle \mathbf{v} \rangle \times \mathbf{H}_{ext}) \cdot d\mathbf{l}]$  along the direction of bias current (Zapata *et al.* [293], Lee *et al.* [294], Wambaugh *et al.* [295]). These predictions are in agreement with recent experimental results obtained for purely superconducting systems (Villegas *et al.* [296], Wördenweber *et al.* [297], van de Vondel *et al.* [298], Togawa *et al.* [299], de Souza Silva *et al.* [300], Wu *et al.* [301], Aladyshkin *et al.* [302]).

### 3.4. Planar S/F bilayer hybrids

In this section we shall discuss the properties of continuous planar S/F structures which have macroscopically large lateral dimensions. As before, the superconducting and ferromagnetic films are assumed to be electrically insulated from each other.

#### *Appearance of vortices in planar S/F structures*

The interaction of the Meissner currents and the currents induced by vortex lines with a one-dimensional distribution of the magnetization (both single domain walls, periodic domain structures and magnetic bars) in the London approximation was considered by Sonin [99], Genkin *et al.* [304], Bespyatykh and Wasilevski [305], Bespyatykh *et al.* [306], Helseth *et al.* [307], Laiho *et al.* [308], Traito *et al.* [309], Erdin [310], Bulaevskii and Chudnovsky [311, 312], Kayali and Pokrovsky [313], Burmistrov and Chtchelkatchev [314], Ainbinder and Maksimov [315], Maksimova *et al.* [316, 317]. It was found that in order to create vortex-antivortex pairs in the S/F bilayer with out-of-plane magnetization at  $H_{ext} = 0$  (and thus keeping the total flux through the superconducting film zero) the amplitude of the magnetization  $M_s$  should overcome the following threshold value [305, 308, 309]

$$M_{v-av}^{\perp} = \frac{H_{c1}}{4\alpha} \frac{D_s}{w} \propto \frac{\Phi_0 D_s}{\lambda^2 w} \ln \lambda / \xi,$$

where  $\alpha$  is a numerical factor of the order of unity and the period  $w$  of the domain structure is assumed to be fixed (the hard-magnet approximation). This estimate corresponds to the case when the width of the domain walls is much smaller than other relevant length scales. The critical magnetization  $M_{v-av}^{\perp}$  decreases monotonically with decreasing superconducting film thickness  $D_s$ . The equilibrium vortex pattern appearing in the superconducting film at  $H_{ext} = 0$  and  $M_s > M_{v-av}^{\perp}$  consists of straight vortices, arranged in one-dimensional chains, with alternating vorticities corresponding to the direction of the magnetization in the ferromagnetic domains [305, 308, 309]. The parameters of such vortex configuration with one or two vortex chains per half-period was analyzed by Erdin [310]. It was shown that in equilibrium the vortices in the neighboring domains are halfway shifted, while they are next to each other in the same domain. Alternatively, as the thickness  $D_s$  increases, the vortex configuration, consisting of vortex semi-loops between the ferromagnetic domains with opposite directions of the magnetization, becomes energetically favorable [308, 309] provided that  $M_s > M_{loops}^{\perp}$ , where

$$M_{loops}^{\perp} = \frac{H_{c1}}{8 \ln(w/\pi\lambda)} \propto \frac{\Phi_0}{\lambda^2} \frac{\ln \lambda / \xi}{\ln(w/\pi\lambda)}.$$

The destruction of the Meissner state in the S/F bilayer with in-plane magnetization was considered by Burmistrov and Chtchelkatchev [314]. Since the out-of-plane component of the field, which is responsible for the generation of the vortex, is maximal near the domain wall (unlike from the previous case) and goes to zero in the center of magnetic domains, one can consider only a single domain wall. At  $H_{ext} = 0$  a creation of a single vortex near the Bloch-type domain wall of width  $\delta$  corresponds to the condition

$$M_v^{\parallel} \simeq \frac{H_{c1}}{4\pi} \frac{\lambda}{D_f} \times \begin{cases} 2\lambda/\delta, & \pi\delta/(4\lambda) \ll 1 \\ 1 - 32\lambda/(\pi^2\delta), & \pi\delta/(4\lambda) \gg 1. \end{cases}$$

#### *Magnetic pinning and guidance of vortices in planar S/F structures*

Irrespective of whether the domain structure in the ferromagnetic layer is spontaneously created or was present beforehand, the spatial variation of the magnetization will lead to an effective vortex pinning (Bespyatykh *et al.* [306], Bulaevskii *et al.* [318]). However, there are discrepancies in the estimates concerning the pinning effectiveness. Indeed, Bulaevskii *et al.* [318] argued that superconductor/ferromagnet multilayers of nanoscale period can exhibit strong pinning of vortices by the magnetic domain structure in magnetic fields below the coercive field when the ferromagnetic layers exhibit strong perpendicular magnetic anisotropy. The estimated maximum magnetic pinning energy for a single vortex in such a system is about 100 times larger than the core pinning energy produced by columnar defects. In contrast to that, Bespyatykh *et al.* [306] have shown that the effectiveness of magnetic pinning of vortices in a layered system formed by an uniaxial ferromagnet, does not considerably exceed the energy of artificial pinning by a column-type defect, regardless the saturation magnetization of the ferromagnet. The limitation of the pinning energy is caused by the interaction of external



vortices with the spontaneous vortex lattice formed in the superconducting film when the magnetization of the ferromagnetic film exceeds the critical value (see Eq. 23).

There have been numerous experimental investigations corroborating the enhancement of the critical current in planar S/F hybrids. It was shown that the presence of a bubble domain structure in Co/Pt ferromagnetic films with out-of-plane magnetization modifies the vortex pinning in superconducting Pb films (Lange *et al.* [205, 256, 319, 320]), leading to an increase of the width of the magnetization loop  $M(H_{ext})$  as compared with a uniformly magnetized S/F sample (Fig. 27). The crossover between an enhanced magnetic pinning on bubble magnetic domains observed at low temperatures and a suppressed magnetic pinning at temperatures close to  $T_c$  for a demagnetized S/F bilayer can be possibly associated with an increase of an effective penetration length  $\lambda^2/D_s$ , characterizing the vortex size, and an effective averaging on the small-scale variation of the nonuniform magnetic field provided that  $\lambda^2/D_s$  considerably exceed the period of the magnetic field (Lange *et al.* [320]). Interestingly, the parameters of the bubble domain structure (the size and the density of domains of both signs of magnetization) can be controlled by demagnetization similar to that reported in Refs. [96, 123]. A three-fold enhancement of the critical depinning current in Nb films fabricated on top of ferromagnetic Co/Pt multilayers was observed by Cieplak *et al.* [321, 322] based on magnetization measurements and on the analysis of the magnetic field distribution obtained by using a 1D array of Hall sensors. The mentioned enhancement of the magnetic pinning takes place in the final stages of the magnetization reversal process, and it can be attributed to residual un-inverted dendrite-shaped magnetic domains.

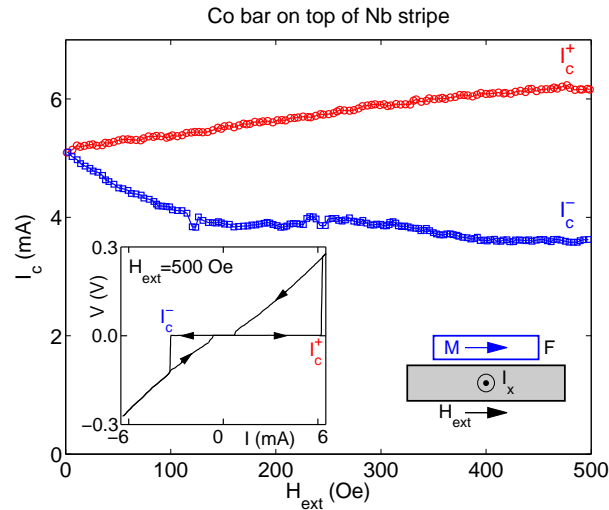
High-resolution magneto-optical imaging performed by Goa *et al.* [323] in superconducting NbSe<sub>2</sub> single crystals and ferrite-garnet films demonstrates that the stray field of Bloch domain walls can be used to manipulate vortices. Indeed, depending on the thickness of the sample, the vortices are either swept away or merely bent by the Bloch wall.

Vlasko-Vlasov *et al.* [247, 324] and Belkin *et al.* [325, 326] studied the anisotropic transport properties of superconducting MoGe and Pb films and NbSe<sub>2</sub> single crystals which are in the vicinity of a ferromagnetic permalloy film. In these works a quasi-one-dimensional distribution of magnetization can be achieved by applying a strong enough in-plane field  $H_{ext} > 300$  Oe, which aligns the domain walls in a desired direction. Such a domain structure was maintained even after switching off the external magnetic field. Magneto-optical measurements directly display the preferential direction of the vortex entry in the presence of the perpendicular magnetic field along the domain walls [the panel (c) and (d) in Fig. 23]. By reorienting the magnetic domains using combinations of dc and ac fields, it is possible to rearrange current patterns in the S/F bilayer and thus manipulate its conductivity. The presence of this rotatable periodic stripe-like magnetic domain structure with alternating out-of-plane component of magnetization results in a difference in the critical depinning current density between cases when the magnetic domain stripes are oriented parallel and perpendicular to the superconducting current:  $J_c^{\parallel} > J_c^{\perp}$ . For planar thin-film Pb/Py structures Vlasko-Vlasov *et al.* [324] observed a pronounced magnetoresistance effect yielding four orders of magnitude resistivity change in a few millitesla in-plane field. In addition, the S/F bilayer exhibits commensurability features that are related to the matching of the Abrikosov vortex lattice and the magnetic stripe domains (Belkin *et al.* [326]). The matching effects are less apparent than for S/F hybrids with magnetic dots, although commensurability becomes more pronounced as temperature is lowered. This result can be explained by the gradual decrease in the  $\lambda$  value, which leads to stronger modulation of the magnetic field in the superconductor at lower temperatures and consequently, to more prominent magnetic interaction with ferromagnetic domain structure.

It is interesting to note that the effect of magnetic domains on the pinning of vortices, was also observed in high- $T_c$  superconductors such as YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$</sub>  (García-Santiago *et al.* [327], Jan *et al.* [328], Zhang *et al.* [329], Laviano *et al.* [330]). At the same time the influence of the ferromagnet on the nucleation in the high- $T_c$  superconductors should be rather small due to extremely small coherence length (of the order of few nanometers).

#### *Current compensation effect and field-polarity dependent critical current*

A superconducting square with in-plane magnetized ferromagnet on top was proposed by Milošević *et al.* [331] as a potential field and current compensator, allowing to improve the critical parameters of superconductors. Indeed, such a magnet generates stray fields of the same amplitude but opposite signs at the poles of the magnet, therefore the field-compensation effect leads to the enhancement of the upper critical



**Figure 28.** (color online) Diode effect in the Nb bridge ( $2\ \mu\text{m}$  width) with the in-plane magnetized Co stripe on top: Experimental dependence of  $I_c^+$  (the critical current in the  $x$ -direction, see the geometry of the S/F system on the inset) and  $I_c^-$  (the critical current in the opposite direction) on the external magnetic field  $H_{\text{ext}}$  applied in the  $y$ -direction ( $T = 4.2\ \text{K}$ ,  $T_{c0} = 9.2\ \text{K}$ ), adapted from Vodolazov *et al.* [333]. In the inset the dc  $I - V$  characteristic of our hybrid system  $H_{\text{ext}} = 500\ \text{Oe}$  is presented, showing a pronounced diode effect.

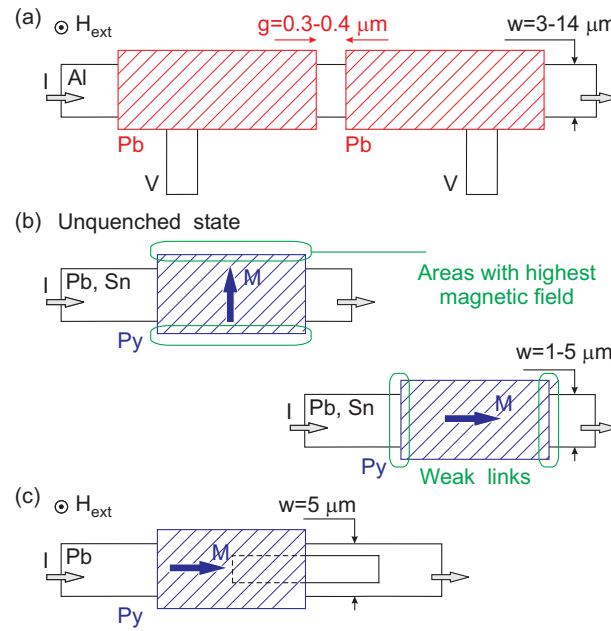
field equally for both polarities of the external field. The superconducting state was shown to resist much higher applied magnetic fields for both perpendicular polarities. In addition, such ferromagnet induces two opposite screening currents inside the superconducting film plane (in the perpendicular direction to its magnetization), which effectively compensates the bias current, and therefore superconductivity should persist up to higher applied currents and fields. These effects have been recently studied experimentally by Schildermans *et al.* [332] in an Al/Py hybrid disk of  $1.7\ \mu\text{m}$  diameter where a finite dipolar moment lying in the plane of the structure was achieved by pinning magnetic domains with the contact leads used for electrical measurements.

Vodolazov *et al.* [333] and Touitou *et al.* [334] considered an alternative experimental realization of the current compensator, consisting of a superconducting bridge and a ferromagnetic bar magnetized in-plane and perpendicularly to the direction of the bias current. Such geometry allows one to weaken the self-field of the superconducting bridge near its edge and thus to enhance the total critical current corresponding to the dissipation-free current flow. Since the self-field compensation occurs only for a certain direction of the current (for fixed magnetization), the presence of magnetized coating leads to a diode effect – the current-voltage  $I - V$  dependence becomes asymmetrical (Fig. 28). Later the similar difference in critical currents flowing in opposite directions was studied experimentally by Morelle and Moshchalkov [335] for a system consisting of a superconducting Al strip placed close to a perpendicularly magnetized Co/Pd rectangle and Vodolazov *et al.* [336] for Nb/Co bilayer in the presence of tilted external magnetic field.

### 3.5. Stray field-induced Josephson junctions

Josephson junctions consist of weak links between two superconducting reservoirs of paired electrons. Commonly, these junctions are predefined static tunnel barriers that, once constructed, can no longer be modified/tuned. In contrast to that, a new concept of the Josephson junctions with a weak link generated by the local depletion of the superconducting condensate by a “magnetic barrier” from a micro/nano-patterned ferromagnet can be realized (Sonin [98]). Interestingly, this type of devices offer an unprecedented degree of flexibility as it can be readily switched ON/OFF by simply changing between different magnetic states using an in-plane field. This switching process is fully reversible, and non-volatile since does not require energy to keep one of the magnetic states.

A pioneer investigation of the properties of superconducting weak links achieved by local intense magnetic fields was performed by Dolan and Lukens [337]. The sample layout used by these authors and their typical



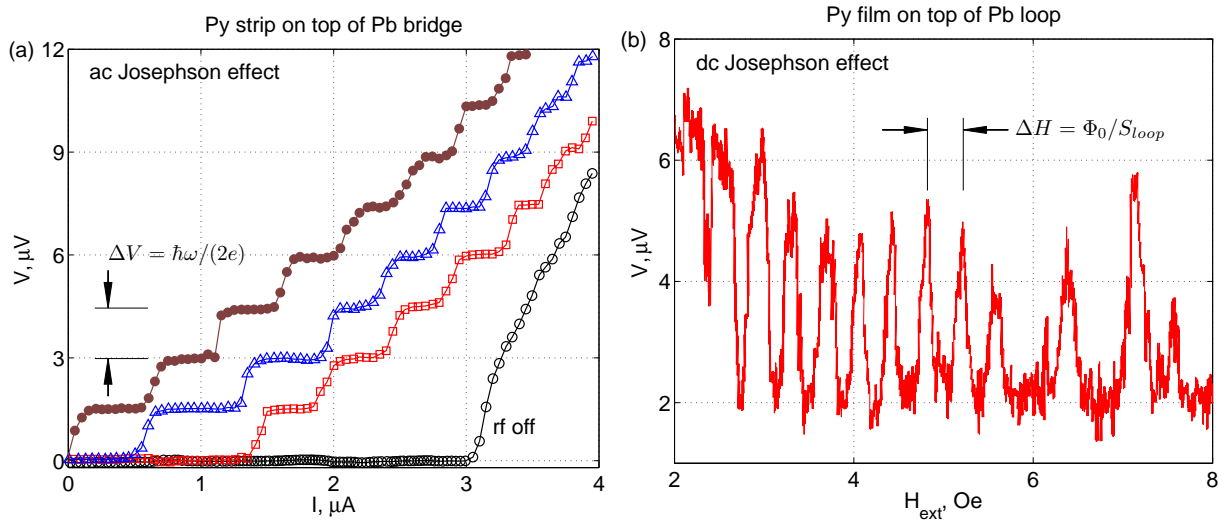
**Figure 29.** (color online) (a) Sample layout investigated by Dolan and Lukens [337]: An uniform Al bridge was covered with a superconducting Pb strips (dashed rectangles) everywhere but in a small region near the center of the bridge. Due to the flux expulsion from the Pb strips the local magnetic field is primarily confined to this gap.

(b) Sample configuration investigated by Clinton and Johnson [338, 339]: a Pb (or Sn) transport bridge is partially covered with a ferromagnetic Py strip with in-plane magnetic moment  $M$ . When  $M$  is parallel to the bridge a strong stray field depletes the superconducting order parameter in a small region near the border of the Py bar (quenched state) thus inducing a weak link.

(c) Schematic presentation of magnetoquenched superconducting quantum interferometer, consisting of two superconducting Pb bridges connected in parallel and permalloy film on top, after Eom and Johnson [342].

dimensions is schematically shown in Fig. 29 (a) and it consists of an Al bridge locally covered by a plain Pb film which has a thin gap of width  $g$  and spans the width of the Al strip at its center. By applying an external magnetic field, the Pb film screens the magnetic field due to the Meissner effect in the whole Al bridge but magnify its intensity at the gap position. This effect leads to a local region of suppressed superconductivity which gives rise to dc and ac Josephson effect as evidenced by a finite critical current and the presence of Shapiro steps in the current-voltage characteristics at  $V_n = n\hbar\omega/(2e)$  when the system was irradiated with rf-excitations with frequency  $\omega = 2\pi f$ ,  $n$  is integer. Interestingly, the Josephson-like features appear for applied fields in the shield gap approximately equal to the upper critical field of the Al film.

An alternative method to obtain a field-induced weak link has been more recently introduced by Clinton and Johnson [338, 339, 340, 341]. The basic device consist of a bilayer of a thin superconducting strip and a ferromagnetic layer with in-plane magnetic moment overlapping the width of the bridge [see panel (b) in Figure 29]. When the magnetic moment is parallel to the superconducting bridge the dipolar fringe is strong enough to locally suppress the superconducting order parameter across the bridge (quenched state) and thus create a weak link. This effect can be turned off by simply magnetizing the ferromagnetic layer perpendicular to the transport bridge with an external in-plane dc field or by a current pulse in a separate transport line [341]. Clearly, the proposed switchable Josephson junction seems to be very attractive for potential technological applications, since energy is required only to change the magnetic states, which are thereafter maintained in thermodynamic equilibrium. Later on based on the same idea Eom and Johnson [342] proposed a switchable superconducting quantum interferometer consisting of a ferromagnetic Py film partly covering two parallel superconducting Pb bridges fabricated in a loop geometry. The dependence of the voltage  $V$ , induced on this superconducting loop at injection of stationary bias current, on the perpendicular magnetic field  $H_{ext}$  is shown in Fig. 30(b) and it reminds the standard Fraunhofer diffraction pattern (Barone and Paterno [34]).



**Figure 30.** (color online) (a) The  $I - V$  curves obtained for a plain superconducting Pb bridge ( $2 \mu\text{m}$  wide) subjected in the inhomogeneous magnetic field, quenched state [see the panel (b) in Fig. 29] for different intensities of rf-irradiation, adapted from Clinton and Johnson [339]. The experiment was carried out at  $T = 5 \text{ K}$ ,  $T/T_{c0} \simeq 0.76$ ,  $H_{\text{ext}} = 0$ , frequency  $f$  of the radio signal equal to  $0.75 \text{ GHz}$ . (b) The  $I - V$  dependence obtained for a superconducting Pb loop of the width  $4.5 \mu\text{m}$  with rectangular hole  $1.5 \times 7.0 \mu\text{m}^2$  covered by permalloy film [see the panel (c) in Fig. 29], adapted from Eom and Johnson [342]. This curve demonstrates the short period oscillations with the period determined by the area of the superconducting loop  $S_{\text{loop}}$ .

#### 4. Hybrid structures: superconductor – soft magnets

Thus far, we have discussed the influence that a ferromagnet has on the superconducting properties of S/F hybrids, assuming that the magnetization of the ferromagnet  $\mathbf{M}$  remains practically unaltered. In this last section, we consider the possibility that the magnetization  $\mathbf{M}$  can be changed either due to the external magnetic field or by the superconducting screening currents induced by the magnetic subsystem, which are particularly relevant at low temperatures. This situation could, in principle, be achieved by using paramagnetic materials or soft ferromagnetic materials with a low coercive field.

The equilibrium properties of “superconductor – soft magnet” hybrid structures (so-called soft S/F hybrids) can be obtained phenomenologically by the minimization of the Ginzburg-Landau energy functional Eq. (1) or the London energy functional Eq. (17), in which the term  $G_m$  responsible for the self-energy of the ferromagnet becomes important

$$G_m = \frac{1}{2M_s^2} \int_{V_f} \left( J_{\parallel} |\nabla M_x|^2 + J_{\parallel} |\nabla M_y|^2 + J_{\perp} |\nabla M_z|^2 \right) dV - \int_{V_f} 2\pi Q M_z^2 dV, \quad (23)$$

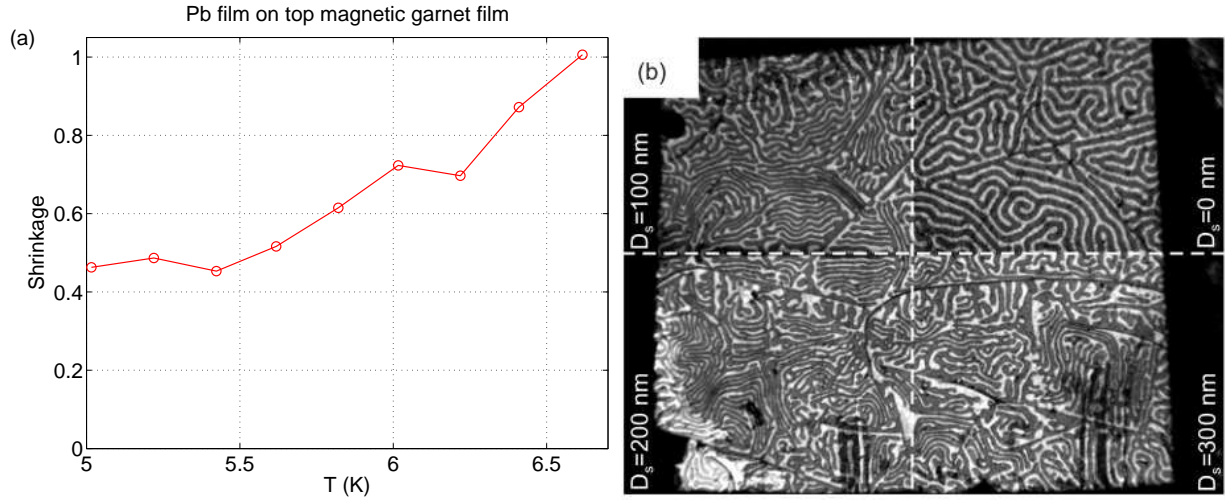
where  $J_{\parallel}$  and  $J_{\perp}$  characterize the exchange interaction between spins in a uniaxial ferromagnet with respect to the in-plane and out-of-plane direction,  $Q$  is a quality factor taking into account the internal anisotropy of the ferromagnet and determining the preferable orientation of the magnetization (either in-plane or out-of-plane). Equation (23) describes the energy cost for having a slowly varying spatial distribution of the magnetization† and, in particular, it describes the energy of a domain wall in a ferromagnet. In some cases (for instance, for rapid  $\mathbf{M}$  variations typical for ferromagnets with domain walls of rather small width), in order to simplify the problem, the increase of the free energy given by Eq. (23) can be taken into account phenomenologically by substituting  $G_m$  by a fixed term  $G_{dw}$  representing the energy of a domain wall.

##### *Modification of the domain structure in a ferromagnetic film by the superconducting screening currents*

The influence of superconducting environment (both substrate or coating) on the equilibrium width of magnetic domains in ferromagnetic films was considered theoretically by Sonin [99], Genkin *et al.* [304], Sadreev [344], Bespyatykh *et al.* [345, 346], Stankiewicz *et al.* [347, 348], Bulaevskii and Chudnovsky [311, 312], Daumens and Ezzahri [349]. In particular, one can expect a prominent change in the equilibrium period of a one-dimensional domain structure at  $H_{ext} = 0$  for rather thick ferromagnetic films ( $D_f \gg w$ ) with out-of-plane magnetization. Indeed, the Meissner currents, induced by the ferromagnet, will decrease the magnetic field inside the superconductor (usual flux expulsion effects) and significantly increase the magnetic field inside the ferromagnet. As a consequence, the density of the free energy of the ferromagnet, given by  $\mathbf{B}^2/8\pi - \mathbf{B} \cdot \mathbf{M}$  or, equivalently, by  $\mathbf{H}^2/8\pi - 2\pi M_z^2$ , raises for a given  $M_z$  distribution. However, the total energy of the S/F system can be lowered by a decrease of the period of the ferromagnetic domains: the smaller the period, the faster the decay of  $\mathbf{H}$  away from the surfaces of the ferromagnetic film. Thus, it is expected that the equilibrium width of magnetic domains in planar S/F bilayer becomes smaller below the critical temperature of the superconducting transition as compared with the state  $T > T_{c0}$ . In contrast to that, for thin ferromagnetic films ( $D_f \ll w$ ) the opposite behavior is predicted: the domain width in the free ferromagnetic film should be smaller than that for the same film on top of a superconducting substrate [348]. This can be understood by taking into account the change of the far-zone demagnetizing field characteristics. In addition, Stankiewicz *et al.* [348] argued that the effect of the superconducting substrate on the period of domain structure in ferromagnetic films with in-plane magnetization is rather small as compared with that for the out-of-plane magnetized ferromagnets. However, an increase of the magnetostatic energy of the S/F hybrids at  $T < T_{c0}$  due to the Meissner currents results also in a shrinkage of the equilibrium width of an isolated 180° Bloch wall, separating two ferromagnetic domains with in-plane magnetization, in the vicinity of the superconducting substrate, as was predicted by Helseth *et al.* [307].

The foreseen decrease of the period of the domain structure in a ferromagnetic garnet film in contact with a superconducting Pb film was recently investigated by magneto-optical imaging (Tamegai *et al.* [350]). It was demonstrated that the shrinkage depends both on temperature and the thickness of superconducting coating

† The theory of superconductor – soft ferromagnet systems near the “ferromagnet–paramagnet” transition has been considered by Li *et al.* [343] within Ginzburg-Landau formalism.



**Figure 31.** (color online) (a) The temperature dependence of the shrinkage ratio of the width of magnetic domains in a hybrid structure consisting of a Pb film ( $D_s = 300$  nm) on top of a magnetic garnet film in comparison with the same ferromagnetic film without superconducting coating, adapted from Tamegai *et al.* [350].

(b) The magneto-optical image of four segments of the superconducting Pb/garnet film structure, differing by the thickness of the Pb film ( $D_s = 0$  nm, 100 nm, 200 nm, 300 nm in a counter-clock-wise direction), taken at  $T = 5.0$  K, adapted from Tamegai *et al.* [350].

layer. The temperature dependence of the shrinkage factor  $s$  evaluated by comparing the average width  $\langle w \rangle$  of the magnetic domain width in regions with and without the superconducting Pb film is shown in Fig. 31(a). It points out that the lower the temperature, the narrower the magnetic domains are ( $s = 0.47$  at  $T = 5.0$  K).

#### *Alteration of magnetization of ferromagnetic dots by the superconducting screening currents*

The Meissner currents also influence the magnetic states and the process of magnetization reversal in ferromagnetic disks placed above a superconductor. It is well known that a uniformly magnetized (single domain) state is energetically favorable for radii  $R_f$  smaller than some threshold value  $R_f^*$  (for a given thickness of the dot  $D_f$ ), while the magnetic vortex state can be realized for  $R_f > R_f^*$ . The typical  $M(H_{ext})$  dependence for ferromagnetic disks for  $R_f > R_f^*$  was already shown in Fig. 25. The dependence  $R_f^*$  vs.  $D_f$  (the phase diagram in the “diameter–height” plane) in the presence of a bulk superconductor, characterized by the London penetration depth  $\lambda$ , was investigated numerically by Fraerman *et al.* [351] and later analytically by Pokrovsky *et al.* [352]. It was shown that the smaller  $\lambda$ , the smaller the critical diameter  $R_f^*$  becomes for a given dot’s thickness. The transitions between the two magnetic states can be induced also by increasing the external magnetic field: the magnetic vortex, possessing an excess energy at zero field, becomes energetically favorable for finite external fields (the magnetic-vortex nucleation field). Although the energy of the interaction between the superconductor and ferromagnet is expected to be much smaller than the self-energy of the ferromagnetic particle (for realistic  $\lambda$  values), it could lead to an experimentally observable decrease in the magnetic vortex nucleation field  $H_{nucl}^{\parallel}$  and increase in the magnetic vortex annihilation field  $H_{ann}^{\parallel}$  (Fig. 32).

The appearance of a spontaneous magnetization of individual S/F hybrids, consisting of an Al bridge and demagnetized Ni dots on top, upon cooling through the superconducting transition temperature at  $H_{ext} = 0$ , was reported by Dubonos *et al.* [353]. Indeed, the reshuffling of magnetic domains in the submicron ferromagnetic disk, caused by temperature-dependent screening of the domain’s stray fields by the superconductor, can explain the observed appearance of nonzero magnetization of the ferromagnet at low temperatures. More recently, the modification of the magnetic state of Nb/Co and Nb/Py superlattices induced by screening currents in the superconducting Nb films was studied experimentally by Monton *et al.* [290, 354, 355] and Wu *et al.* [356].

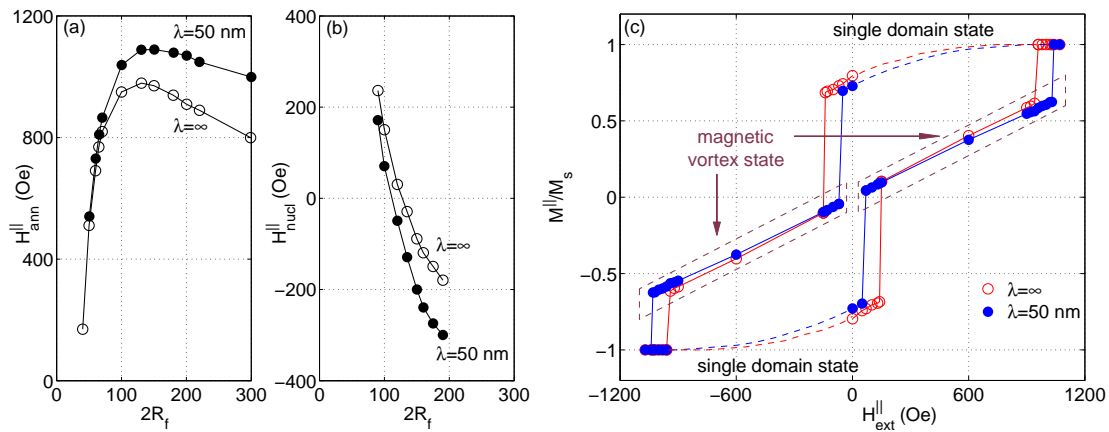
Kruchinin *et al.* [273] demonstrated theoretically that a superconducting environment modifies the magnetostatic interaction between localized magnetic moments (embedded small ferromagnetic particles), resulting either in parallel or antiparallel alignment of neighbor dipolar moments at  $H_{ext} = 0$ . The crossover

between these regimes depends on the ratio of the interparticle spacing and the London penetration depth, and thus preferable “magnetic” ordering (ferromagnetic vs. antiferromagnetic arrangements) can be tuned by varying temperature.

#### Mixed state of soft S/F hybrid structures

The magnetostatic interaction between a vortex-free superconducting film and a uniformly magnetized ferromagnetic film at  $H_{ext} = 0$  may cause the spontaneous formation of vortices in the superconductor and magnetic domains in the ferromagnet in the ground state of planar S/F bilayers with perpendicular magnetization. Lyuksyutov and Pokrovsky [357], and Erdin *et al.* [358] argued that the ground state of the S/F system could be unstable with respect to the formation of superconducting vortices. Indeed, for a uniformly magnetized S/F bilayer, characterized by a magnetization of the ferromagnetic film per unit area  $m = M_s D_f$ , the magnetostatic interaction between the superconductor and the ferromagnet changes the total energy of an isolated vortex line to  $\varepsilon_v = \varepsilon_v^{(0)} - m\Phi_0$  [146] as compared with the self-energy of the vortex in the superconducting film  $\varepsilon_v^{(0)}$  without a ferromagnetic layer. As a consequence, the formation of vortices becomes energetically favorable as soon as  $\varepsilon_v < 0$  (either for rather large  $m$  values or at temperatures close to  $T_{c0}$  where  $\varepsilon_v^{(0)}$  vanishes). However, as the lateral size of the S/F system increases, the averaged vortex density  $n_v$  would generate a constant magnetic field  $B_z \simeq n_v \Phi_0$  along the  $z$ -direction which can lead to an energy increase larger than the gain in energy due to creation of vortices. Hence, in order for the vortex phase to survive, the ferromagnetic film should split in domains with alternating magnetization in a finite temperature range at  $T < T_{c0}$ . As long as the magnetic domain width exceeds the effective penetration depth, the energy of the stripe domain structure seems to be minimal (Fig. 33). Interestingly, the interaction between a single vortex in a superconducting film and the magnetization induced by this vortex in the adjacent ferromagnetic film can cross over from attractive to repulsive at short distances (Helseth [171]).

Carneiro studied the interaction between superconducting vortices and a superparamagnetic particle with constant dipolar moment, which is assumed to be able to freely rotate, in the London model [162, 163, 164]. It was found that, due to the rotational degree of freedom, the pinning potential for superconducting vortices differs significantly from that for a permanent dipole. In particular, the interaction between the superconducting



**Figure 32.** (color online) (a) The dependence of the magnetic vortex annihilation field  $H_{ann}^{\parallel}$ , corresponding to the transition from the magnetic vortex state to a single-domain state, on the diameter of disk  $2R_f$ , calculated for an isolated ferromagnetic disk of 20 nm thickness (i.e., without superconductor,  $\lambda = \infty$ , open circles) and for the same disk placed above a bulk superconductor ( $\lambda = 50$  nm, filled circles), adapted from Fraerman *et al.* [351].

(b) The dependence of the magnetic vortex nucleation field  $H_{nucl}^{\parallel}$ , corresponding to the transition from single-domain state to the magnetic vortex state for the same problem, adapted from Fraerman *et al.* [351].

(c) The magnetization curve  $M^{\parallel}/M_s$  vs. in-plane external field  $H_{ext}^{\parallel}$  demonstrating the process of the magnetization reversal for the magnetic disk (20 nm thickness and 100 nm diameter) for  $\lambda = \infty$  (open circles) and  $\lambda = 50$  nm (filled circles), adapted from Fraerman *et al.* [351]. Thus, the screening effect increases the width of the  $H_{ext}$  interval at the ascending and descending branches of the magnetization curve where magnetic vortex state is energetically favorable.

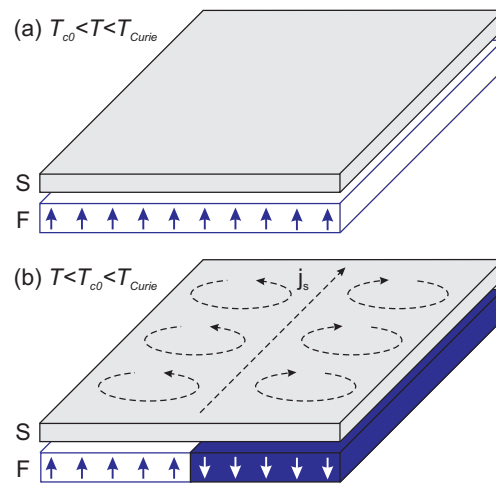


vortex and the magnetic dipole can be tuned by applying an in-plane external field: the corresponding depinning critical current was shown to be anisotropic and its amplitude potentially varies by as much as one order of magnitude. Later on, this approach was generalized by Carneiro [255] for hybrid systems consisting of thin superconducting film and a soft ferromagnetic disks in the magnetic vortex state (similar to that for Refs. [351, 352] but considering a vortex line inside the superconducting sample).

A new method of pinning vortices in S/F epitaxial composite hybrids consisting of randomly distributed Gd particles incorporated in a Nb matrix was reported by Palau *et al.* [275, 274]. Since the size of Gd particles are much smaller than the coherence length and the interparticle distance is much shorter than the penetration depth, this regime of collective magnetic pinning differs both from conventional core and magnetic pinning mechanisms. In this case, since a vortex “feels” a homogeneous superconductor (for length scales on the order of  $\lambda$ ), pinning effects are expected to be small. However, due to the local field of a vortex, the Gd particles can be magnetized and a moving vortex would lead to hysteretic losses in the magnetic particles, which in turn results in an increased pinning (for decreasing magnetic fields).

#### *Superconductor – paramagnet hybrid structures*

An alternative way of modifying the superconducting properties of soft hybrid structures is by using paramagnetic constituents, characterized by zero or very low remanent magnetization. Such superconductor-paramagnet hybrids with a magnetization  $\mathbf{M} = (\mu - 1) \mathbf{H}/4\pi$  depending on the external field ( $\mu$  is the magnetic permeability) in the presence of transport current were considered theoretically by Genenko [359], Genenko and Snezhko [360], Genenko *et al.* [361, 362, 363] for  $\mu \gg 1$ . It was predicted that the paramagnetic material placed near superconducting stripes and slabs can drastically modify the current distribution in such hybrids, thus, suppressing the current enhancement near the superconducting sample’s edges inherent for any thin-film superconductor in the flux-free current-carrying state. As a consequence, the current redistribution leads to an increase of the threshold value of the total bias current corresponding to the destruction of the Meissner state. In other words, the magnetically shielded superconductors even in the Meissner state are able to carry without dissipation rather high transport current comparable with the typical current values for a regime of strong flux pinning [359, 361, 362, 363]. A survival of the Meissner state for thin-film superconducting rings carrying a current and placed between two coaxial cylindrical soft magnets was studied by Genenko *et al.* [364, 365] and Yampolskii *et al.* [366]. The similar problem concerning with the distribution of magnetic field inside and outside a superconducting filament sheathed by a magnetic layer, as well as the magnetization of such a structure in the region of reversible magnetic behavior in the Meissner state was considered by Genenko *et al.* [367]. The formation of the mixed state in various superconductor/paramagnet structures in the presence of transport



**Figure 33.** (color online) (a) Uniform state of the S/F bilayer above the superconducting critical temperature, (b) Spontaneously formed magnetic domain structure and coupled chains of superconducting vortices with alternating vorticity at  $T < T_{c0}$ , adapted from Erdin *et al.* [358]. Solid arrows correspond to the magnetization of the ferromagnet, while black arrows schematically show the circulating superconducting currents.



current, or an external magnetic field  $H_{ext}$  or the field of hard magnetic dipoles, were analyzed by Genenko *et al.* [368], Genenko and Rauh [369], Yampolskii and Genenko [370], and Yampolskii *et al.* [371, 372, 373] in the framework of the London model. The Bean-Livingston barrier against the vortex entry in shielded superconducting filaments was shown to strongly depend on the parameters of the paramagnetic coating and, as a result, the critical field at which the first vortex enters can be enhanced [368].

Since for the superconductor/paramagnet hybrids there are no changes neither in the vortex structure in the Meissner state of superconductor or in the magnetic state of paramagnetic elements, characteristics of superconductor-paramagnet hybrids are presumed to be reversible, and ac losses should be minimal for such structures. It stimulated the implementation of paramagnetic and ferromagnetic coatings in high- $T_c$  superconductors in order to improve the critical current and reduce the ac-losses (Majoros *et al.* [374], Glowacki *et al.* [375], Horvat *et al.* [376, 377], Duckworth [378], Kovac *et al.* [379], Touitou *et al.* [334], Pan *et al.* [380], Jooss *et al.* [381], Gu *et al.* [382], Gömöry *et al.* [383, 384]).

Although, strictly speaking, permalloy is a ferromagnet with rather low-coercive field, it can behaves qualitatively similar to paramagnetic materials. Indeed, the magnetization vector for thin-film structures deviates from in-plane orientation if a perpendicular external field is applied. Such rotation of the magnetization of the dot toward the out-of-plane direction while sweeping the external field gives rise to a  $z$ -component of magnetization depending on the external field. The effect of the stray field generated by soft permalloy dots on the critical current  $I_c$  of the superconducting Al loops was experimentally studied by Golubović and Moshchalkov [385]. The monotonous decrease in the period of the oscillation on the  $I_c(H_{ext})$  with increasing the  $H_{ext}$  value was interpreted as a flux enhancement due to the increase of the out-of-plane component of the dot's magnetic moment. In this sense soft magnetic materials are promising candidates for the design of a linear magnetic flux amplifier for applications in superconducting quantum interference devices.

## 5. Conclusion

We would like to conclude this work by formulating what, we believe, are the most exciting unsolved issues and possible interesting directions for further studies of S/F hybrid systems.

*Spontaneous formation of a vortex lattice and domain patterns.* As we discussed in Section 4, the magnetostatic interaction between a vortex-free superconducting film and a uniformly and perpendicularly magnetized ferromagnetic film at zero external field, may lead to a spontaneous formation of vortex-antivortex pairs in the superconductor accompanied by alternating magnetic domains in the ferromagnet in the ground state (see Fig. 33 and Refs. [33, 358]). To the best of our knowledge, thus far there are no experimental results confirming this prediction. In part, this is likely due to the difficulties of combining a very low coercive field ferromagnetic materials, needed to guarantee the free accommodation of magnetic domains, and a well defined out-of-plane magnetic moment.

*Magnetic pinning.* Based on London equations we have clearly defined the magnetic pinning energy as  $\mathbf{m}_0 \cdot \mathbf{B}_v$  (see section 3). Since London equations are valid as long as core contributions are negligible, in principle this simple relationship holds for materials with  $\kappa = \lambda/\xi \gg 1$  and low temperatures. Unfortunately in the vast majority of the experimental reports so far, it seems that these conditions are not satisfied. Any other contributions such as local suppression of  $T_c$ , proximity effect, or local changes in the mean free path, which are not accounted for within London approximation, could lead to a deviation from the treatment in the framework of London model. The problem that remains unsolved so far is the identification of the most relevant mechanisms of vortex pinning in S/F hybrid systems.

*Thermodynamic properties of S/F heterostructures.* Although the electric transport in superconductor/ferromagnet hybrid systems has been intensively studied during the past decades, very little is known about their thermodynamic and thermal properties such as their entropy, specific heat, thermal conductivity, etc. From an academic point of view it would be very relevant to investigate the nature of the phase transitions, or present thermodynamic evidence of confinement of the superconducting order parameter. On the other hand, estimating the heat released when the system changes its state might also provide useful information for devices based on S/F heterostructures.

*Electromagnet-superconductor hybrids.* Most of the research performed so far has been focussed on the effects of an inhomogeneous field generated by a ferromagnetic layer onto a superconducting film. As was early demonstrated by Pannetier *et al.* [97] in principle there is no difference whether this inhomogeneous field is the stray field emanating from a permanent magnet or the magnetic field generated by micro (nano) patterned current carrying wire on top of the surface of the film. The great advantage of the latter is the degree of flexibility and control in the design of the magnetic landscape and the external tuneability of its intensity. Such electromagnet-superconductor hybrids represent a promising alternative for further exploring the basic physics behind S/F hybrid.

*Time resolved vortex creation and annihilation.* Josephson  $\pi$ -junctions give rise to spontaneous formation of half-integer flux quanta, so called semifluxons (Hilgenkamp *et al.* [386]). It has been theoretically demonstrated that for long Josephson junctions with zigzag  $\pi$ -discontinuity corners the ground state corresponds to a flat phase state for short separation between corners whereas an array of semifluxons is expected for larger separations. Interestingly, by applying an external bias current it is possible to force the hopping of these semifluxons between neighboring discontinuities (Goldobin *et al.* [387]). This hopping of semifluxons could be identified through time resolved measurements with drive amplitudes above the depinning current. There are clear similarities between these arrays of semifluxons in zigzag Josephson systems and the vortex-antivortex arrays in S/F systems. Indeed, recently Lima and de Souza Silva [292] have shown theoretically that the dynamics of the vortex-antivortex matter is characterized by a series of creation and annihilation events which should be reflected in the time dependence of the electrical field. Experimental work corroborating these predictions are relevant for understanding the dynamics of flux annihilation and creation in S/F systems.

All in all, the strive to comprehend the ultimate mechanisms ruling the interaction between ferromagnets and superconductors has made this particular topic an active theoretical and experimental line of research. We believe that these vigorous efforts will inspire further developments in this area of solid state physics and

perhaps motivate new applications of technological relevance.

## Summary of experimental and theoretical researches

	Co/Pt Co/Pd	BaFe <sub>12</sub> O <sub>19</sub> PbFe <sub>12</sub> O <sub>19</sub>	Co, Fe, Ni	Fe/Ni (Py)	Other ferromagnets
<b>S/F hybrids consisting of large-area superconducting and plain non-patterned ferromagnetic films (single crystals) with domain structure</b>					
Pb films	[205, 256, 319, 320] [388]	[103]		[324]	[350]
Nb films	[105, 106, 110, 321] [322, 392]	[101, 102] [104]	[290, 354, 355, 389] [393, 394, 395, 396]	[108, 356]	[276, 390, 391]
Al films	[96, 107]				
Other low- $T_c$ films	[397, 398]		[271, 376, 377, 380]	[100, 247] [325, 326]	[109, 323]
High- $T_c$ films	[328, 334]	[327]	[374, 375, 379, 382] [383, 384, 399]	[399]	[329, 330, 378] [381, 400, 401] [402, 403]
<b>S/F hybrids consisting of large-area superconducting film and ferromagnetic elements: single particles, periodic arrays of magnetic dots (antidots) and stripes</b>					
Pb films	[121, 198, 199, 200] [201, 203, 204, 205] [206, 232, 234, 235] [236, 246, 404, 405]		[196, 197, 198, 199] [200, 201, 202, 205] [237, 238, 257, 286] [287, 288, 303, 406]		
Nb films	[280, 281, 282, 284] [283]		[184, 185, 186, 187] [188, 189, 190, 191] [192, 193, 194, 195] [240, 245, 248, 249] [250, 296, 407, 411] [412, 413, 414, 415] [416, 417]	[97, 254] [407, 408]	[119, 120, 263] [274, 275, 276] [277, 409, 410]
Al films	[96, 107, 123, 124] [125, 418, 419]		[215, 253, 303]	[244, 258, 259]	[278]
Other low- $T_c$ films	[397]		[211, 260, 261, 262] [272, 279]		
High- $T_c$ films			[285, 420]		[155, 156, 158] [174, 175, 176]
<b>Laterally confined and mesoscopic S/F hybrids</b>					
Pb films			[340, 421]	[339, 340, 341] [342]	
Nb films			[333, 336]	[341]	
Al films	[133, 134, 135, 136] [137, 140, 335, 405] [423, 424]		[353]	[332, 385, 422]	[337]
Other low- $T_c$ films			[272, 367]	[338]	

**Table 1.** Summary of experimental research on vortex matter in the S/F hybrids with dominant orbital interaction (suppressed proximity effect).

	<b>Ginzburg–Landau formalism</b>	<b>London formalism</b>	<b>Newton–like description of vortex dynamics</b>
Bilayered and multilayered large-area S/F structures (superconducting ferromagnets)	[50, 93, 94, 95] [96, 105, 106, 343] [425]	[98, 99, 146, 173, 270, 304] [305, 306, 307, 308, 309, 310] [311, 312, 313, 314, 318, 344] [345, 346, 347, 348, 349, 357] [358, 426, 427, 428, 429, 430] [431, 432, 433, 434, 435, 436] [437]	
Individual F elements over large-area S films (inside bulk superconductors)	[51, 54, 63, 76] [127, 128, 129]	[146, 147, 148, 149, 150, 153] [154, 155, 156, 159, 160, 161] [162, 163, 164, 165, 166, 167] [168, 169, 170, 171, 172, 173] [255, 273, 278, 291, 351, 352] [431]	
Arrays of F dots elements over large-area S films (inside bulk superconductors)	[112, 113, 114, 115] [116, 117, 118, 122] [124, 142, 143, 182] [183, 419]	[187, 225, 226, 227, 228, 229] [230, 243, 420, 404, 431]	[231, 243, 291] [292, 303, 404] [416, 417, 438]
Laterally confined and mesoscopic S/F structures	[40, 54, 126, 130] [131, 132, 134, 135] [137, 138, 139, 140] [144, 145, 331, 423]	[151, 152, 267, 268, 269, 315] [316, 317, 333, 439]	
Hybrid structures with paramagnetic elements		[359, 360, 361, 362, 363, 364] [365, 366, 367, 368, 369, 370] [371, 372, 373]	

**Table 2.** Summary of theoretical research on vortex matter in the S/F hybrids with dominant orbital interaction (suppressed proximity effect)

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